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Job Search and Null Offers; An analysis
of the causes and consequences of offer rationing
in labour markets.

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Summary:

This thesis examines the consequences and causes of offer rationing in labour markets characterised by job search. Offer rationing is incorporated by allowing for a probability that search is unsuccessful this being referred to as the 'Null Offer' probability. In the first instance the consequences of offer constraint for the job search decisions of individuals are examined. Results existing in the literature are reviewed and then extended to the case where individuals are allowed to learn about the degree of rationing that they face. Secondly the ways in which offer rationing may arise out of the profit maximising decisions of firms are considered. In the absence of an appropriate literature the first task is to specify an appropriate framework for analysis which is subsequently used to examine the factors that might affect the null offer probability. Finally the consequences for market equilibrium of the existence of offer rationing is examined. It is shown that the existence of null offers opens up the possibility of multiple equilibria entailing different unemployment rates and levels of social welfare.

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Chapter 1

Introduction

1.1. Preliminaries

The purpose of this thesis is to provide an analysis of the consequences and causes of offer rationing in labour markets characterised by search. The notion that search activity might be an important feature of labour markets was first discussed in a seminal paper by Stigler (1962). Subsequently a vast literature has developed examining for the most part individual job search decisions (see Lippman and McCall (1976) or Chalkley (1982) for a survey) but also suggesting the consequences of search for market (equilibrium) outcomes. In some of this literature explicit allowance is made for the fact that individuals may face the possibility of unsuccessful search via a probability of failure.

This notion of offer 'rationing' in search is used throughout this thesis in an attempt to subject to greater analysis than thus far attained the causes and consequences of quantity constraint on agents' behaviour and market outcomes. We henceforth refer to the probability of failure in a search context as the probability of a null offer. Searching for a job and being unsuccessful can usefully be thought analogous to soliciting a zero offer. Similarly for a firm, turning away a potential employee who has contacted the firm is analogous to making an unacceptable or null offer.

The following chapters can be divided into three parts. In chapters 2 and 3 the consequences of the existence of null offers for individual decision making in a search context are examined. Chapter 2 essentially reviews the literature on an individual's job search decisions as it relates to the consequences of null offers, whilst chapter 3 extends existing analysis by considering the effects of learning in a 'rationed' setting.

In chapters 4 and 5 the object of analysis is a consistent model of a firm's behaviour. With this aim in mind we concentrate on a firm's wage and vacancy decisions in a market characterised by search and examine ways in which null offers may arise. The existing literature on firm's decisions is very sparse and hence both chapters 4 and 5 represent original contributions. Chapter 4 sets the stage by considering the appropriate framework for analysis whilst chapter 5 presents a model where null offers by firms are a realistic outcome.

In chapter 6 we draw together the analysis of the previous four chapters to examine an equilibrium model of search, unemployment and null offers. The aim here is to examine ways in which the existence of imperfect information and job search might change the usual notion of an efficient equilibrium in a competitive market.

In the remainder of this chapter, each of the above three sections of the thesis is considered in more detail. The methods and objectives of analysis are of particular concern.

1.2. Models of Individual Search Behaviour

It is now some 20 years since Stigler (1962) first considered the problem of job choice in a market characterised by imperfect information. Although Stigler (1962) considered many elements of a job to be associated with uncertainty ("stability of employment, conditions of employment") he concentrated attention on uncertainty regarding the location and value of available wages. Subsequent writers (e.g. McCall (1965,1970) extended this lead and considered the correct mathematical specification of a model allowing for search

by individuals. Since then considerable research activity has been devoted in generalising models of individual search behaviour and it is these generalisations as applied to a model of search that allow for offer rationing that chapter 2 will review.

Throughout the process of formulating and then generalising a model of individual job search decisions interest has centred around unemployment. It is assumed throughout that the activity of searching for a job can be most effectively conducted whilst unemployed. The attraction of a model that in part explains the role of unemployment in a labour market is obvious. It has long been noted that turnover is an important feature of labour markets and it was towards an explanation of labour market flows that the model of individual job search was first applied. It is a considerable step to go from accepting that job search activity is an element explaining the economic role of unemployment spells of individuals to a model of the economy that allows for no other cause of unemployment. Many of these early attempts at 'explaining' unemployment via simple models of individual job search (see Phelps et al (1970)) have subsequently been seen to be misconceived (see Lippman and McCall (1976a)). In any case a model that deals only with one side of the market is subject to the criticism that its predictions may not be valid when the operation of the whole market is considered. The difficulties are most acutely observed in Rothschild (1973), but are also realised in many early contributions (see Alchian (1970), Gronau (1973)).

It is the contention here that models of individual search decisions should not be applied to make general inferences about the operation of markets characterised by job search. Rather they serve essentially two roles.

First they provide an input into market equilibrium models of search. This role will be seen in later chapters of this thesis (see chapter 6). Such are the complexities of market models that only the simplest specifications of individual job search behaviour will be used.

Secondly, models of individual decision making are useful precisely because they cast light on the determinants of individual decisions. In this role search theory can be seen as an extension to the basic labour supply literature where the extension involves allowing for uncertainty. Clearly some allowance for the fact that individuals may face constraints as regards to the offers that it is possible to obtain is an important ingredient of such a model. Offer rationing has indeed been an essential ingredient of attempts to empirically estimate models of individual job search behaviour (see Nickell (1979)). The uncertainty involved in locating wage opportunities is interesting because it invokes an active response from individuals. It is this element of active response to lack of information that we feel makes models of individual decision making interesting in their own right and particularly rich in explaining many aspects of individual behaviour. One particular kind of active response to uncertainty, namely information acquisition, is singled out for attention in chapter 3 where a model of search incorporating rationing is generalised to allow for learning about the extent of rationing.

Chapter 2 starts by considering the simplest possible search problem involving offer rationing. In such a setting it is only the location and not the value of vacancies that is unknown to individuals. The next step is to build up the 'stylised' model of job search that has been the main concern of the job search literature.

The central results of this literature are then explained in relation to the null offer model of job search. Throughout the exposition attention will be drawn to the way in which refinements of the basic model add to an understanding of aspects of labour market behaviour and affect the predictions of the basic job search model.

This leads naturally to chapter 3 where the null offer model of job search is extended in a way not previously considered. The rationing of offers that an individual faces is assumed to be learnt over the course of search. Whilst general 'learning' has been considered an important aspect of search activity by individuals, it is not generally possible to solve analytically the optimal search strategy. Indeed, often the nature of the decision problem is so complex that even numerical methods can only be employed at considerable cost (for examples see Hey (1981)). However, learning about rationing of offers turns out to be a considerable simplification over general learning processes. Whilst an analytic expression cannot be found for the reservation wage (the single choice variable determining search strategy) it is relatively easy to solve the model numerically and examine the optimal strategy under a variety of circumstances. The learning model of chapter 3 serves a number of purposes. First it is shown that the predictions from a static (no learning) search model are in many ways quite robust to such a generalisation. Secondly, and more interestingly, the model can be used to examine the effect of 'subjective' uncertainty on search behaviour. It is shown that a generally held result that greater subjective uncertainty leads to lengthier search does not hold (and in fact the reverse is true) if previously examined offers cannot be returned to. Thirdly the richer nature of a more general search model is illustrated

by suggesting possible applications of the learning model to labour market phenomena such as worker discouragement.

Having identified in chapters 2 and 3 the basic ingredients of what has become a large literature on individual search decisions, attention is next centred on models of firms behaviour consistent with job search and in particular offer rationing.

1.3. Models of Firms Decisions, Wage Determination and Offer Rationing

By comparison with the vast literature on individual search decisions, the literature relating to consistent 'demand side' explanations of search markets is scant indeed. Only recently has any attempt to model firms behaviour been made (see Eaton and Watts (1977) and McKenna (1980)). This relative neglect seems in retrospect rather strange, particularly given the tendency of the early search literature (cf Phelps (1970)) to model unemployment as an equilibrium search phenomena. The issues raised by the existence of job searchers in the labour market as regard to their consequences for firms decisions are many and interesting in their own right. As with models of individual decision making, it is possible to identify two roles for models of firm behaviour.

First, any model of market equilibrium must consider explicitly decisions made by agents on both sides of the market. Again it might be expected that the complexities involved in considering a market equilibrium lead to only the simplest models of a firm's behaviour being considered.

Secondly, models of a firm's behaviour provide valuable insights into the nature of the determinants of wages and vacancy creation in

markets characterised by search. It is particularly interesting to ask in this context what sort of model is consistent with the notion of offer rationing that forms the basis of the models of individual decision making in chapters 2 and 3.

There are at least two aspects of the job search process that give rise to important consequences for an analysis of firms decisions.

First imperfect information on the part of individuals gives a firm operating in the labour market monopsony power. A small decrease in the offered wage will no longer result in an immediate and permanent reduction in employment to zero. A particular question of interest here is whether for homogenous labour and identical firms, this monopsony power leads to a non-degenerate distribution of wage offers as an equilibrium outcome (see Braverman (1980) for a discussion of this same question in a price search setting). The answer to this question, however, requires analysis of an equilibrium market model of search. Our analysis therefore attempts to explain how various parameters of the search process (such as the frequency with which individuals contact the firm) affect the monopsony wage decision.

Secondly, the fact that workers search (and are subject to uncertainty regarding the actual location of different wage offers) means that a firm's employment level follows a stochastic process (i.e. a probabilistic process evolving over time). It is this second feature of the search process that leads to the greatest difficulty in modelling firms' decisions and in our view has been largely responsible for the comparative neglect of this area. The problem of modelling decisions in the face of a stochastic labour supply is particularly acute because in a job search context it is the probabilities of various outcomes that a firm in part determines.

Consider by way of example a firm operating in a labour market in which individuals search randomly for jobs. Decisions of the firm regarding vacancy creation, wage offers, job advertising and recruitment screening standards may all affect the probabilities of one or more individuals leaving or joining the workforce. Attempts to model problems such as this in full detail are almost certainly doomed given current techniques. Indeed, even fairly restricted attempts to model certain aspects of this kind of problem have met with only limited success in generating an economic model that is acceptably easy to analyse (see Eaton and Watts (1977)).

We therefore concentrate in chapters 4 and 5 on just two aspects of firms decisions - those regarding wages and employment. If wages are viewed as being determined by firms rather than by market forces it becomes relevant to ask what factors influence a firm's wage decision and in which direction.

In chapter 4, therefore, an approach to modelling such problems is suggested and applied to several simple wage determination problems. The novelty of the approach lies in considering the process of arrivals to and departures from a firm occurring continuously through time. This naturally leads to consideration of stochastic processes for a firm's employment level that can adjust only by 'unit' changes instantaneously. Such stochastic processes are considerably easier to analyse than more general Markov processes and this fact is reflected in the literature on such processes (see Cox and Miller (1970)). The important point to note about the analysis of chapter 4 is that the existence of job search renders the assumption of wage taking behaviour implausible. Given this, the determinants of a firm's profit maximising

wage offer are examined. It is shown that the existence of a distribution of reservation wages across searching individuals poses serious problems for analysis. It becomes very difficult to sign the effects of parameter changes even for highly simplified models. It is expecting too much to imagine that the simple models outlined in Chapter 4 will provide a rigorous and usable explanation of, for example, wage dispersion. Indeed it is shown in chapter 4 that the influences on wage determination even within a very simple search setting are sufficiently complex as to render unambiguous comparative statics results impossible. Instead therefore the following general conclusions are drawn from the analysis. If firms do indeed unilaterally set wages in a market characterised by search the following factors will be a priori important;

1. The rate at which individual searchers contact the firm
2. The extent of natural turnover and quitting
3. The distribution of characteristics over applicants.

All of these factors will in turn be influenced by labour market conditions.

The models of firms' behaviour discussed in chapter 4 either consider a firm to create an infinite number of vacancies or, at the other extreme, only a single vacancy. However, the method of modelling of firms' decisions outlined in that chapter can be applied to a more general model where the actual number of vacancies created by a firm which faces some vacancy cost function is a decision variable. The exposition and analysis of such a model is the concern of chapter 5. A model which is general enough to allow for a finite number of vacancies to be created and which can demonstrate the phenomenon of a firm giving a null offer is clearly an important part of any

consideration of the causes of quantity constrained search. In previous work in this area the generation of 'null' offers has either been ignored (by effectively assuming that firms always have vacancies) or assumed (by positing single vacancy firms). The existence of a finite bound on vacancy creation that is a decision variable of the firm complicates the analysis of firm's decisions. This being the case, numerical analysis is used in chapter 5 to suggest some comparative statics results where such effects cannot be signed analytically. Of particular interest is the way in which a firm's decisions determine the probability that the firm is found in a state of 'no vacancies' and it is on this particular aspect of the model that attention is focused. Exogenous changes that lead to either an increase or decrease in the above probability are discussed. An issue of interest in the analysis of chapter 5 is the possibility of a feedback loop between individuals' and firms' decisions. It can be shown, for example, that increased search activity may generate more vacancies and less rationing, which in turn is consistent with more search activity.

In both chapters 4 and 5 firms are generally assumed to monopsonistically determine wages given an upward sloping expected employment function. In the final section of chapter 4 the ways in which the general approach to modelling of firms' employment state can be adapted to allow for wage bargaining between worker and firm are discussed. The bargaining solution to wage determination in job search settings has found favour in the analysis of Diamond (1982) and Pissarides (1984) and can be successfully integrated into the continuous time stochastic employment model of a firm that is developed in these two chapters.

It should be clear that chapters 2, 3 and 4, 5 respectively discuss the supply and demand sides of markets characterised by search and offer rationing. It remains to consider the operation of such markets as a whole.

1.4. Equilibrium Models of Search with Offer Rationing

It has long been realised that inferences made from partial equilibrium models may prove completely wrong at a general equilibrium level. This is as true for a single market as it is for a system of markets. The dangers of treating an analysis of one set of agents' decisions as a theory of market outcomes are succinctly outlined in Rothschild (1973). Rothschild refers to much of the search literature as "partial-partial equilibrium" analysis, since in general only one side of one market is considered.

The problem of making incorrect inferences from partially specified models is of course a problem of simultaneity. Where one set of agents' decisions depend on market outcomes and where all agents' decisions affect those outcomes, the interdependencies of such decisions must be explicitly accounted for. It is the exploration of this interdependency of decisions towards which market equilibrium models of search are directed. It should not be surprising therefore that generalising our model of search in one direction frequently requires simplification in another. Only the simplest formulations of individual and firm decision problems can be assumed in an equilibrium setting.

The existing literature on market equilibrium models of search is sparse, the earliest paper being that of Lucas and Prescott (1974). More recently, several writers have considered equilibrium models that contain features of the search problem that are of central concern here (Diamond (1982), Pissarides (1979),(1984), Diamond and Maskin (1979)).

There are essentially two types of questions to which answers can be attempted in an equilibrium setting, those concerned with a description of the outcome and those concerning its desirability.

First, how equilibrium outcomes change with changes in the environment (as represented by changes in the exogenous parameters of the model) and in particular what inferences can be drawn about the effect of government action on the operation of markets characterised by search. In a model with offer rationing it seems appropriate to ask in this context what affects the extent of rationing.

The idea that a given set of parameter values is consistent with only one (unique) equilibrium configuration need no longer hold in an imperfect information environment. Where multiple equilibria are possible a further area of interest is with regard to a comparison of the different equilibria.

Secondly, because the interdependency of agents' decisions is explicitly modelled it is possible to ask whether or not the market outcome is 'efficient' or 'optimal'. The usual definition of efficiency is that of an outcome, where no agent can be made better off without simultaneously making some other agent worse off. The notion of 'optimality' allows (given some weighting function for agents' welfare) the distributional effects as well as allocative effects of market outcomes to be evaluated.

Chapter 6 is directly concerned with answering both these types of questions. As noted earlier, the complexities involved in modelling a market equilibrium suggest that only the simplest formulation of decision problem for the various agents be employed.

Chapter 6 starts by outlining the components of an equilibrium model of search. Underlying such a model are models of firms and individual decision making. The purpose of chapter 6 is to explicitly determine the role of offer rationing in a search framework, therefore two specifications are suggested, one where vacancies are always created (no rationing) and one where all firms are limited to a single vacancy (null offers). It is shown that when offers are rationed, non unique equilibria are possible and the qualitative features of these equilibria are examined. Different equilibria will exhibit different levels of wages, unemployment and offer rationing. This is contrasted with the unrationed world where a unique natural rate of unemployment is shown to exist.

Finally, chapter 6 also discusses the welfare implications of search equilibria. Here the analysis of Diamond (1982) suggests a general inefficiency of search equilibria brought about by private wage bargains that fail to reflect the value of entering agents in a matching process.

1.5. Methods

The method and tools of analysis used throughout this thesis are those common to economic theory. A mathematical model of economic behaviour is derived and the consequences for behaviour of changes in parameters analysed by conventional calculus, dynamic programming or other appropriate techniques. Where analytic results are not possible, numerical simulation is used in order to generate results. Numerical methods and computational techniques are detailed in the Appendix.

Throughout this work there is an assumption that agents maximise their objective functions. The 'costs' of computing optima (either in time or resources) are ignored, but this is a feature of the vast majority of the economics literature.

Chapter 2

Null Offers and the Individual's

Job Search Decisions

2.1. Introduction

It is usual in models of individual search decisions to start by assuming a distribution of wage offers existing in the labour market. The optimal search strategy with respect to this distribution is then derived and typically the implications of such a strategy for observable behaviour discussed. Discussion of the problem of search was initiated by Stigler's (1962) pioneering work. The realisation that the optimal strategy frequently assumed a sequential form (i.e. involved the observation of and decision about a sequence of wage offers) followed the work of McCall (1970).

In this chapter we take a somewhat different approach as the object is to review the results of the literature as they pertain in particular to the search problem of an individual facing offer constraint.

In the next section a simple model of 'vacancy' search is outlined in which a single valued wage offer is available to the individual at unknown locations (firms). The only feature of the market that affects an individual's behaviour is the possibility that no offer (i.e. a 'zero' or 'null' offer) is obtained at any stage. Since the only 'gain' to search is the possibility of obtaining a job and the only 'costs' the expenditure on acquiring offers and the waiting cost of unemployment, the search strategy for the individual involves simply a decision as to whether search is at all worthwhile.

In the following sections the above model is generalised to allow for a distribution of wage offers (known or unknown), the effects of finite wealth or a finite planning horizon, the effects

of differing attitudes to risk, alternative uses of time and finally the possibility that jobs may vary in characteristics other than simply the wage. These generalisations effectively review the literature on individual job search decisions by application to the problem of search given offer rationing. At each stage the consequences of search decisions for the observable behaviour of the individual are discussed.

In the concluding section of this chapter the central results of search theory are again briefly repeated and some suggestions made as to the ways in which the issue of offer rationing can be further explored within models of individual search decisions. One such extension, namely an allowance for learning about offer rationing is the concern of Chapter 3.

2.2. Search for a Vacancy

It is useful to have in mind a base model to which more complicated formulations can be related. The essential feature that is the focus of attention in the present work is the notion of job offer rationing so we start with a model that has this as its only feature.

Consider an individual who is currently unemployed and receiving some level of income b per period. b may be in the form of central government unemployment benefits or income from past savings, etc. We assume throughout that from a state of unemployment an individual can search for a job. For present purposes it suffices to assume that all jobs are identical and pay a wage of \bar{w} per period. Let the cost of searching out a single firm be c and let at the most one such

search take place per period.¹ Since not all firms contacted will have a vacancy we assume a probability q of contacting a firm which will actually make a job offer. If the individual is completely uninformed² as to the location of vacancies, we might expect random search over firms and that q represents the proportion of firms with a vacancy. $(1-q)$ is the probability of unsuccessful search and is the probability that the individual receives a 'null' or zero offer. Within the constraints of the current setting the individual only has a choice as to whether to search or not and we start by examining the determinants of this decision. We make three further simplifying assumptions in the first instance, i) that a job once obtained is never lost ii) that the individual lives for ever and iii) that wealth is infinite so that payments of c can continue forever (search can be conducted indefinitely).

The most useful and general approach to the analysis of job search decisions is via the methods of Dynamic Programming (see Bellman (1957)). Assuming that the individual maximises his/her discounted lifetime earnings and has discount factor ρ ($\rho = 1/(1+\zeta)$, where ζ is the individual's rate of time preference), the value of search can be written as

$$(2.1) \quad V_s = b - c + [qV_E + (1-q)V_s]\rho$$

where V_s is the expected discounted lifetime income of a searching

-
1. Throughout this and the next chapter a 'discrete' time approach is considered; this is purely a matter of convention in the literature and convenience. Subsequently when a continuous time formulation appears more useful (in Pt. 3) we shall reconsider the vacancy search model in continuous time.
 2. As will be seen in chapters 4 & 5, this idea becomes most plausible when the location of vacancies is constantly changing because of probabilistic influences.

individual and V_E is the value of employment. Clearly under the simplifying assumptions made so far

$$(2.2) \quad V_E = \bar{w}/(1-\rho)$$

The value of search must be compared with the only alternative available to the individual, namely permanent unemployment. Again under the present assumptions this can simply be written as

$$(2.3) \quad V_U = b/(1-\rho)$$

The individual will choose that strategy that maximises expected income, i.e. will choose $\max(V_U, V_S)$.

The infinite horizon assumption means that essentially nothing changes with the passing of time and in particular the 'value' of search or unemployment is constant. We can therefore solve for V_S as

$$(2.4) \quad V_S = \frac{(b-c)}{(1-(1-q)\rho)} + \frac{\rho q V_E}{(1-(1-q)\rho)}$$

which must be compared with V_U . The individual will choose either to search or remain permanently unemployed as $(V_S - V_U)$ is positive or negative. We can therefore solve for a wage w^* that makes the individual indifferent between search for employment and permanent unemployment.

$$(2.5) \quad w^* = \frac{(c-b)(1-\rho)}{\rho q} + \frac{[1 - (1-q)\rho]b}{\rho q} = b + \frac{c}{\rho q} (1-\rho)$$

w^* has a natural interpretation as the participation wage. It is that wage which if available to the individual will just ensure his participation in the labour market.

Notice that since (2.4) is increasing in q that w^* must be decreasing in q . Better employment prospects lead to 'greater' participation.

w^* is clearly also increasing in c and decreasing in b . Greater costs of search lead to less participation. As unemployment benefits rise the value of 'search' increases less quickly than the value of permanent employment and hence participation declines.

In the absence of unemployment benefits the participation wage simplifies considerably. The value of permanent unemployment becomes zero and hence

$$(2.6) \quad w^* = \frac{(1-\rho)c}{\rho q} \Rightarrow w^*q = \zeta c$$

Notice here the role of discounting, the greater is discounting of future returns (the smaller is ρ) the less worthwhile search becomes. The true cost of search therefore has two components, the money cost c and the cost of time.

This highly simplified framework helps to make clear what are the central insights of search theory. The assumption that has been invoked which makes the above analysis distinguishable from the usual analysis of participation decisions is that of imperfect information regarding the location of employment opportunities.

Such imperfect information together with a cost of becoming informed suggests that there is a genuine economic decision to be analysed. Are the benefits of becoming informed, which in this simple model are easily defined as the value of employment itself V_E , sufficient to compensate for the expected costs of acquiring information? The answer of course depends on the magnitude of benefits (namely the wage \bar{w}) the cost of each observation (i.e. C), the number of observations on average required in order to find a job (governed by q), and if time itself is costly the degree of impatience (given by ρ).

Within the framework suggested here it is easy to relax the three assumptions made earlier. (For simplicity b is set equal to zero.)

Rather than assume an infinite duration job we could allow for a constant risk per period μ of job loss. In this case the value of search is as before [(2.4) with $b = 0$] but the value of employment must be written as

$$(2.7) \quad V_E = w + \rho [\mu V_S + (1-\mu)V_E]$$

We therefore have a pair of simultaneous equations in V_E and V_S to solve for the participation wage. From inspection, however, it is clear that (since $V_E > V_S$) w^* will be increasing in μ . An increased risk of job loss results in a lower participation rate through the increase implied in the costs of search.

The imposition of a finite planning horizon either through myopia or finite wealth considerations means that we can no longer analyse 'stationary' value functions, indeed we now need to 'date' both V_E and V_S .

Denoting by $V_S(t)$ the value of search when t periods remain,

$$(2.8) \quad V_S(t) = -c + [qV_E(t-1) + (1-q)V_S(t-1)]\rho$$

$$V_E(t) = w + \rho V_E(t-1) = \frac{w}{(1-\rho)} \quad \text{since} \quad V_E(t) = V_E(t-1)$$

In this case the value of search must be calculated recursively from the horizon (T) backwards, using the fact that $V_S(0) = 0$. By simple recursion it follows that $V_S(t) > V_S(t-1) \forall t$ and hence that if an individual finds it worthwhile to make the last search (s)he will find it worthwhile to search whenever possible.

This simple model also allows for the possibility that an individual becomes discouraged during the course of search. If $V_S(t)$ declines to zero before the horizon is reached (where $V_U(t) = 0 \forall t$) then the individual will permanently abandon search or drop out of the labour market.

Various other simple extensions to the model of 'vacancy' search outlined here are of course possible. The opportunity of obtaining more than one vacancy and therefore of searching more 'intensively' can easily be incorporated into this model, as can the possibility of searching selectively from markets with 'different' q 's. In the latter case an obvious result is that those markets with the best prospects (highest q 's) will be searched first. Both of these extensions to the simple model will again be considered when wage dispersion is allowed for in the next section.

A more interesting possibility is to allow for the probability q to be unknown by the individual and learnt during the course of search. This sort of adaptive search model and generalisations of it to the case where a distribution of wage offers is available is the concern of the next chapter.

It is useful, however, to pause to consider the implications of the above simple model for individuals' behaviour in a labour market context. We have outlined a model in which the individual chooses the 'state' to be occupied given a choice between non-participation and active search for a vacancy. Considerable insight is gained from formal analysis of the problem even in as simple a setting as the one above. In particular the expected returns and costs of actions determine behaviour and in a stationary world actions once determined will not evolve over time. Where a unique wage offer is available the returns from search take the form of the expected income (suitably discounted) when a job is found, that is

the discounted wage weighted by the probability in each period of actually finding a vacancy or soliciting a non-zero offer. These returns must be balanced against the costs of search which come in a direct money form (c), and in the form of impatience. In particular if the individual has the opportunity of some income now (b) of which some may be foregone in order to increase income in the future, impatience or 'discounting' is an important consideration.

Even within this simple framework we therefore have a model that suggests an explanation of an individual's (labour market) state occupancy decisions. Whilst empirically it may be very difficult to distinguish individuals by the two states examined (which may both appear as unemployment), economic theory has furnished us with a set of variables which might be relevant in explaining any observations. The particular variable of interest in this analysis is the probability q which has played a central role in the model to date. It is worth repeating the significance of this variable. The probability of obtaining an offer (or equivalently the probability of failure $(1-q)$) dictates the expected waiting time to a job offer. Since waiting is costly both in time (foregoing income now for returns in the future) and in direct costs (c) the offer probability ' q ' affects the individual's search decision trade-off.

The model of vacancy search developed here has not received such a comprehensive treatment in the job search literature which instead, following the lead of Stigler (1962), has concentrated on the consequences of wage dispersion. Assuming a distribution of wages

can be seen as a generalisation of the vacancy search model developed here and it is a generalisation that will be discussed in detail in the next section. The consideration of wage dispersion concentrates attention on an individual's decisions whilst in the 'state' of searching. In particular an important question becomes how to characterise the optimal search strategy. Of course a complete treatment of the search problem involves both elements, which state to occupy and how to search given that it is desirable.

What such an approach suggests is that job search theory at the individual level should provide an explanation of individual behaviour with respect to three states, Non Participation, Search Unemployment and Employment and, in particular, the determinants of the transitions between these states. Again it should be noted that in this work we are particularly concerned with an examination of the effects of offer rationing in the labour market as reflected in q . So far we have examined the choice between the occupation of the first two of these states with respect to the role of q . It is now time to expand our analysis.

2.3 Wage Dispersion

Assume now that the wage offers available to the individual are given by a continuous distribution of wage offers $f(w)$ (the following arguments apply equally to cases of discrete distributions) and that the three assumptions of the last section hold. We can now view the individual's decision problem as viewed from a state of unemployment as a two stage one. Firstly, whether it is worthwhile to search at all which depends on whether the 'value' of search is greater than the value of non-participation (zero in the case of $b = 0$).

Secondly, how much to search given that search is desirable, which decision we might expect to depend upon the marginal valuation of costs and benefits of search.

We have already examined the first decision in some detail and will henceforth concentrate mainly on the second. The approach taken to this second decision by Stigler (1962) was that of determining the optimal sample size, i.e. number of searches to be made. This, however, is a sub-optimal way for the individual to behave since it ignores the essential sequential nature of the problem analysed by McCall (1970) and subsequent writers.

Consider, then, the value of job search for an individual faced by a distribution of offers and having just received a draw on that distribution of \bar{w} . We assume here that offers once rejected cannot be returned to³ so that the value of search may be written ($b=0$)

$$(2.9) \quad V_s(\bar{w}) = \max\left\{ \frac{\bar{w}}{(1-\rho)}, -c + \left[q \int_0^u V_s(w) f(w) dw + (1-q)V_s(0) \right] \rho \right\}$$

The value of search must be the maximum of two alternatives, to stop now and quit search (which yields $\bar{w}/(1-\rho)$) or to search again. u is the upper bound of $f(w)$.

Looking at these two alternatives as a function of \bar{w} we notice that the value of the first increases monotonically with \bar{w} whilst the second is a constant. This immediately suggests that there will be a unique value of \bar{w} that equates the two expressions and that

3. This is the 'no recall' assumption which seems quite plausible in many labour market situations. In the present case the ability to return to past offers does not affect decisions; where it does both this and the case of perfect 'recall' will be allowed.

this value divides the set of all possible offers into those which are worth more than further search ($w > r$) and those that are worth less ($w < r$). r is thus sufficient to entirely capture the individual's search decisions in this setting. It is this remarkable feature that has led to a great deal of research effort in determining under what generality of assumptions such a property will continue to hold. r has a natural interpretation as a 'reservation' wage that is the wage that makes an individual indifferent between accepting it or continuing search. The optimality of a reservation wage strategy in a search problem gives it the so-called reservation property. The literature that fully discusses this property is immense, the most general treatments being in the literature on statistical decisions DeGroot (1970), Chow et al (1971), Brieman (1964). Proofs of the existence of the reservation property in job search settings are to be found in Lippman and McCall (1976a), Kohn and Shavell (1974), Landsberger and Peled (1977).

The reservation wage for the problem under consideration here can be implicitly defined by noting that

$$(2.10) \quad \frac{r}{(1-\rho)} = -c + \rho [qE[V_s(w)] + (1-q)V_s(0)]$$

Following a reservation wage strategy implies therefore an expected value of search given by $E[V_s(w)] = E \max \left[\frac{w}{(1-\rho)}, \frac{r}{(1-\rho)} \right]$

where E denotes the mathematical expectation.

The value of search when no offer has been obtained $V_s(0)$ is clearly therefore $\frac{r}{(1-\rho)}$.

Since w is a random variable distributed according to $f(w)$ it follows that

$$(2.11) \quad E \max \left[\frac{w}{(1-\rho)}, \frac{r}{(1-\rho)} \right] = \frac{r}{(1-\rho)} \int_0^r f(w)dw + \int_r^u \frac{w}{(1-\rho)} f(w)dw$$

which can be rearranged to give

$$(2.11a) \quad \frac{1}{(1-\rho)} E \max (w, r) = \frac{1}{(1-\rho)} \left[r + \int_r^u (w-r)f(w)dw \right]$$

Substituting into (2.10) we have

$$(2.12) \quad r + c.(1-\rho) = (q[r + \int_r^u (w-r)f(w)dw] + (1-q)r)\rho$$

which gives us an implicit equation for r of the form

$$(2.13) \quad c(1-\rho) - q\rho \int_r^u (w-r)f(w)dw - (1-\rho)r = 0$$

This is precisely Lippman and McCall's (1976a) equation (15) only with w and r representing a per period payment rather than lifetime earnings streams.⁴

Equation (2.13) or formulations similar to it form the basis for by far the greater part of the literature on individual search decisions.

4. To clarify notation equate ξ to $\frac{r}{1-\rho}$, β to ρ and x (on offer) to $\frac{w}{1-\rho}$.

A number of results are apparent at once, namely that r is decreasing in c and increasing in ρ (that is decreasing in the discount rate). The effects of changes in the offer distribution $f(w)$ can also be signed for certain cases. An increase in the risk (in the Rothschild-Stiglitz (1970) sense) of $f(w)$ increases r for example (see Lippman and McCall (1976a)).

Our parameter of interest is q and this affects r positively. Thus we have identified two consequences of offer rationing for individual search decisions. In the first section it was noted that the parameter q was important in determining whether or not participation or 'search' was worthwhile, here we have a model for examining the role of q in determining the actual quantity of search (in expectation) that an individual will engage in.

The first consideration of a search problem where offers might be 'rationed' via some probability of search success is to be found in Baron (1975) and subsequently Feinberg (1977), this type of model is included in the survey of Lippman and McCall (1976a). One way of viewing the q parameter is as a component of the 'full' offer distribution facing the individual. Denoting by $g(w)$ the 'full' distribution then,

$$\begin{aligned} (2.14) \quad g(w) &= (1-q) \quad w = 0 \\ &= qf(w) \quad \forall w > 0 \end{aligned}$$

In this sense an increase in the probability of failure can be seen as an unambiguous worsening of the full offer distribution which causes search to become less valuable to the individual.

In the case of our model with a single wage we could only make statements regarding the state occupancy of an individual; with an analysis including dispersion it is possible to suggest how behaviour within a state (namely search unemployment) is determined.

Where (as has been assumed so far) the horizon of the individual is infinite and the parameters of the problem stationary through time, the reservation wage will be a constant and gives us in part the probability that in any period the individual will exit from the 'search' state into the employed state (ρ_s). The probability of receiving an offer in excess of r is simply $(1-F(r))$ where $F(w)$ is the cumulative distribution function of wage offers. It therefore follows that the transition probability ρ_s is given by

$$(2.15) \quad \rho_s = q(1-F(r))$$

It has long been noted (Baron (1975), Feinberg (1977) and Nickell (1979)) that an increase in q has an ambiguous effect upon ρ_s . Whilst offers arrive more frequently the individual is more choosy and therefore may exit the state of search unemployment with a smaller probability. It can be shown, however, that for 'reasonable' specification of $f(w)$ $d\rho_s / dq > 0$ (Feinberg(1977)).

We can combine the two models of this and the previous section and draw inferences about both the participation and search decisions of the individual. If the reservation wage that solves (2.13) declines to zero, then it is clear that the individual will not find it optimal to participate. It follows that for some value of q the prospects of an offer may be so poor that the expected lifetime earnings of an individual net of search costs become negative, and hence that the worker becomes 'discouraged'.

The existence of 'income' during search in the form of b of course complicates the above analysis somewhat. The intuition, however, remains clear, if the reservation wage (which will be a function of b) is calculated to be less than b , it is optimal for the individual to 'accept' b and not engage in further search. It is important to notice the dual role of b in this case. An increase in benefits unambiguously increases the reservation wage (smaller direct cost of search) but may also increase discouragement. It is usual in the literature to comment on only the first of these two effects.

The model with wage dispersion and 'null' offers can be generalised along the lines of our approach in section 1. It should be clear that if jobs have a risk of unemployment attached to them their 'value' declines as therefore does the reservation wage. The fact that different jobs paying the same wage may have differing unemployment risks does not result in a trade off, however, of wages for security (see Burdett and Mortensen (1980)).

Inclusion of a finite search horizon has exactly the same consequences as in our simple vacancy search model. It becomes necessary to calculate the reservation wage recursively commencing at the horizon. This is the case except if perfect recall is possible (see Lippman and McCall (1976)). In this very special case the reservation wage is time invariant (and equal to the infinite search horizon wage) right up to the horizon. Again, if no recall or only imperfect recall is possible (see Landsberger and Peled (1977), Karni and Schwartz (1977)), the reservation wage will decline as the horizon approaches. Numerical results suggest (see Chalkley (1982a)) that the effect of a finite horizon is not particularly great but that its imposition may result in a reversal of Feinberg's (1977)

results concerning the expected duration of unemployment as a function of q^5 .

The model of search with wage dispersion described so far is highly stylised. A great deal of the literature in this area has been concerned with successive generalisations of this stylised model. The results of this research as it relates to our 'null offer' augmented search model are briefly outlined in the next section.

2.4 Some Extensions

i) Search Intensity

It is a comparatively simple matter to modify the analysis of either of the last two sections to include the possibility of choice as regards intensity of search, provided one regards all offers obtained within a search period as being 'viewed' simultaneously. In the case of our vacancy search model we might posit a cost of (simultaneously) contacting firms of the form

$$(2.16) \quad c = c(n)$$

where n is the number of contacts.

Clearly the gain from search is

$$(2.17) \quad [\text{Min}(1, qn)] \cdot \frac{w}{(1-\rho)}$$

5. Feinberg suggests that the duration of search unemployment is a negative function of q , but Chalkley (1982a) shows that this result may be reversed for 'short' search horizons and small search costs.

and our analysis goes through as before only with an optimal n to solve for subject to search being worthwhile:

A similar modification for the wage search model requires that the expected gain from search be written as

$$(2.18) \quad E[V_g(w)] = qE \max [w, r]$$

where w is an n vector of offers.

If variable search intensity is interpreted as being the 'speed' with which offers are acquired (i.e. that offers are still viewed sequentially), then it is more helpful to consider the search problem in continuous time.

ii) Ordering Prospects

Some writers have considered as important the idea that individuals might be able to order firms according both to the probability that an offer will be given and also as to the distribution from which the offer is drawn. Optimal 'search' (given that any search is worthwhile) in this case entails the determination of an ordering of firms and then a sequential search starting at the best firms first.

A model such as this has been examined by Salop (1973). From the point of view of the present work, the important results concern the effect of differing probabilities of a 'null' offer between different firms. In the case of no wage dispersion (vacancy search)

it is clear that an individual will search firms (if they can be identified) with highest q first. Salop (1973) shows that in the case of wage dispersion a trade off exists between the quality (w) and probability (q) of an offer.

Although not remarked upon by Salop (1973), the above model has another interesting implication regarding individual search decisions in that it allows for a worker to abandon search (become discouraged). It also suggests that better informed job searchers (those able to order prospects) will search less (see Gera and Hasan (1980)).

iii) Search in a Dynamic Context

The consideration of search prospects varying across firms leads naturally to a consideration of search strategies when prospects vary through time.

The simplest possibility is to allow for a set of q 's or $f(w)$'s that will face the individual through time and to assume that these are known with certainty by the individual. In this case the solution of the optimal search problem again becomes one of calculating recursively from the horizon the value of search alone (in the case of vacancy search) or given a reservation wage strategy (in the case of wage dispersion). The case of wage dispersion is dealt with by Pissarides (1976b). Allowing for non stationary environments of course leads to richer models. In particular the possibility exists that an individual becomes discouraged for certain 'periods' (when prospects are particularly bad) and then becomes active again as labour market conditions improve. Unfortunately the assumption that the future is so well known is a rather strong one

but relaxing it considerably complicates analysis.

A more general assumption would be that the economy evolves according to some stochastic process, the parameters of which are known to the individual. Lippman and McCall (1976c) examine this possibility in the wage dispersion search model. As noted above, this kind of model is in essence a generalisation of Salop's (1973) work. Except for showing that an optimal search strategy will possess the reservation 'property' few other results are possible within such a general framework.

Given the emphasis here it is useful to consider how dynamic considerations might be included into the simple vacancy search model.

Consider the case of no discounting over the course of search (but where the value of an offer is finite, i.e. $\frac{W}{(1-p)} = W$) and assume now that the state of the labour market is reflected in q which can take on a finite number of values, q_i $i = 1, \dots, n$.

The value of search now of course depends upon the 'state' so that

$$(2.19) \quad V_s(i) = -c + q_i V_E + (1-q_i) \sum_{j=1}^n p_{ij} V_s(j)$$

where p_{ij} is the state transition probability, i.e. (the probability that the q changes from q_i to q_j) and $V_E = \frac{W}{(1-p)}$.

The decision of whether to search or not will depend upon the current state i . Some states will both be directly unfavourable and have a low probability of improvement. In such states the individual may not find it worthwhile to search. Even in states which are currently favourable but which have a high probability

of worsening, the individual might not find it worthwhile to search.

Some states may be 'captive', that is to say can never be escaped from once entered, similarly some states may never be entered and therefore be irrelevant to the problem.

If the stochastic process governing the evolution of q is time varying (and therefore not Markovian), further analysis becomes almost impossible analytically and optimal search strategies will need to be investigated numerically.

iv) Risk Aversion

Most studies of uncertainty or probability in economic theory start with an analysis of attitude to risk (see Rothschild and Stiglitz (1970)). So far, however, it has been assumed that the individual maximises his lifetime wages so that utility is a simple linear function of uncertain income. The incorporation of risk aversion into job search problems is not as straightforward as might appear. Nachman (1975), Hall, Lippman and McCall (1978) and Danforth (1979) consider this problem.

As far as our basic vacancy search model is concerned, risk aversion can be incorporated without difficulty. Following the usual definition (Rothschild and Stiglitz (1970)) a more risk averse individual will have a lower value of search and will therefore be less likely to search. Hall Lippman and McCall (1978) show that analogues of these results hold in the case of search without recall in the wage dispersion model. Allowing for perfect recall, however, causes great difficulties since the optimal strategy need no longer possess the reservation property. Intuitively the reason

for this is that with concave utility high offers received at the start of the search process may be rejected and later recalled if nothing better 'turns up'. The failure of the reservation property in a search context is analogous to dynamic inconsistency as identified in other economic models (see Kydland and Prescott (1977)) and implies similar difficulties in analysis.

v) Learning

So far the environment facing the individual searcher (the full distribution of offers $g(w)$) has been assumed known. The possibility that information may be incomplete and acquired over the course of search opens up interesting areas of analysis.

In the first instance it integrates search theory with 'information economics' where the acquisition of information is seen as having important consequences for economic analysis (see Varian (1978)).

Secondly, allowing for individuals to adapt to their environment is clearly a major step towards realism in search models, where perhaps the perfect information assumptions seem unduly strong.

The analysis of 'optimal' learning (via Bayesian techniques) in search models has been investigated by Rothschild (1974), Kohn and Shavell (1974) and Rosenfield and Shapiro (1981). A dissenting view and advocacy of satisficing models is given in Hey (1981).

The literature has concentrated on the effects of learning about $f(w)$, the distribution of positive offers and the complexity of the decision problem in such environments has led to a paucity of

useful results, useful that is in the sense of providing empirical implications. Indeed, much effort has been expended in examining under what conditions search is well behaved (possesses the reservation property) in a learning environment (see Rothschild (1974) and Rosenfield and Shapiro (1981)). Restricting attention to where the reservation property holds (Kohn and Shavell (1974)) only one 'result' is readily available, the ability to learn if perfect recall is possible leads to greater search. Since, as has been already noted, the perfect recall assumption is a very special case in a labour market context, and not without its problems (when one considers the effect of risk aversion), it might be thought that this approach is rather devoid of content. However, if one is less ambitious and considers particular kinds of imperfect information, greater analysis may be possible. One very obvious candidate for greater investigation is the rationing parameter q . In the next chapter we consider analysing the effect of learning about q over the course of search, and are able to suggest results for the simple vacancy search model and for a model with wage dispersion where $f(w)$ is known.

From the point of view of applicability a model that incorporates learning about offer availability suggests a far more satisfactory explanation of worker discouragement in a search context than has hitherto been offered.

vi) The Opportunity Cost of Search

The textbook analysis of labour supply decisions starts with a definition of utility involving both income and time, in the form of leisure. In such a framework the opportunity cost of searching

is not simply forgone income (c) and impatience (ρ), but also forgone leisure.

A number of writers have therefore attempted to incorporate search decisions into more general time allocation problems, either within a period or over a lifetime. Such a consideration of the opportunity cost of search is not going to be a feature of our further analysis. The interested reader is therefore referred to Baron and McCafferty (1977). McCafferty (1979) and Burdett (1979), who consider allocation of time within the period and Seater (1977, 1979) who considers a life-cycle model of search, work and leisure decisions.

This literature essentially relaxes the assumption that search is an intensive activity (requires a full input of time) which should be considered a maintained (and simplifying) assumption of our analysis.

vii) Characteristics of Jobs

The fact that a 'job' is not simply a wage but many other characteristics was readily acknowledged by Stigler (1962) who chose to concentrate on wages as determining search. The approach of Lancaster (1979) can usefully be employed in once again generalising the notion of a job in terms of the characteristics embodied in it.

If all characteristics can be viewed upon locating a job offer, then our previous analysis generalises relatively easily. The individual might set a reservation utility and accept only jobs embodying characteristics offering at least that utility.

A more reasonable assumption might be that some characteristics can only be experienced (Nelson (1970)) and an analysis of search under such circumstances has been the concern of Hey and Mavomaras (1981), McCall (1980), McCall and McCall (1981), Hey and McKenna (1981) and Wilde (1980, 1979).

Again such generalisations of the notion of a job will not be allowed in the following and therefore the results in this literature are not reviewed.

2.5. Summary and Conclusions

This chapter, by considering a particular kind of search problem where search may be unsuccessful, has reviewed the central results of the literature on an individual's optimal job search decisions.

A model of an individual's search decisions can have two uses.

First, as an input into a market equilibrium model of search, null offers and unemployment where clearly only the simplest type of model, such as that considered in section 2.2 above, will be useful. In this respect we will have cause to use the results of the literature reported here in later chapters.

Secondly, as an explanation of an individual's decision (and therefore predictor of such decisions), when job turnover, unemployment spells and uncertainty regarding the location or value of wage offers are features of the labour market.

In this latter regard we have identified two decisions for which job search theory, where appropriate allowance is made for

rationing of offers, may prove to be a useful and ultimately empirically implementable theoretical foundation.

The first 'decision' is whether any search is worthwhile and the way in which search considerations enter into this decision can be seen by examining a highly simplified model where uncertainty regarding the location of offers is the only feature.

The second decision concerns how much search is to be undertaken contingent upon it being deemed worthwhile. Here the paradigm case offered in the literature (cf. Lippman and McCall (1976a)) where wage dispersion is considered, provides a framework of analysis.

We have seen that in a very simple model the first decision will depend upon a) the money costs of search, b) the opportunity cost of time, c) the level of unemployment income, d) the value of employment, e) the duration (in expectation) of jobs and f) the extent of offer rationing or availability as captured in q .

Where wage dispersion is added, the second decision depends upon all of the above plus the nature of the distribution of wage offers. Where the above models are generalised to include ordering of prospects, risk aversion, changes through time or learning during the course of search, both decisions will depend on these factors as well.

One possible area of development that has been identified in this chapter is the inclusion of learning into search models. Here the existing literature contains few results because of the technical complexity of the issues. However, in a labour market characterised by relatively constant wages (or wage distributions) and quantity adjustment, one might expect individuals to concentrate on

learning about the availability rather than the value of offers. This issue has not so far been addressed in the literature and is the subject of the next chapter.

We conclude this chapter with a comment on the empirical implementation of search theory. Of the two decisions noted above the second (concerning how much search) has been singled out for empirical investigation. In particular researchers have been interested in how government policy instruments influence the quantity of search individuals undertake (see Nickell (1979)). Models of an individual's search decisions cannot of course be used as a basis for welfare judgements regarding the efficiency of search unemployment. However, if one were to make broad statements it might be suggested that participation rather than the quantity of search will have the greater welfare implications. In this case it may be appropriate to devote more resources to the study of participation decisions in a search context, than is currently the case. In this regard the availability, rather than the value of offers may be a crucial determining factor.

Chapter 3

Learning About Null Offers

Section 3.3 of this chapter in a revised form is forthcoming in The Economic Journal (Conference Papers Supplement 1984).

3.1. Introduction

In chapter 2 we reviewed a model of job search that allowed through a probability $(1-q)$ for unsuccessful search on the part of the individual. This probability was seen to be an important determinant of both the participation decision and the quantity of search decision.

Of the generalisations suggested for the basic search model one of the most interesting as regards insight into individual decision making concerns allowance for the fact that some aspect(s) of the individual's environment may be unknown. If this is the case search becomes not just an attempt to locate job offers but also a method of acquiring information. It has long been realised that attempts by economic agents to acquire or transmit information have far reaching implications for the analysis of markets (see Varian (1978) Ch.8). The acquisition of information also poses very interesting questions at a purely individual level. Do better informed individuals search more or less? Is the ability to learn a good or a bad thing? In what ways does the acquisition of information alter the predictions of a model that makes no allowance for learning?

These latter questions are what we shall address ourselves to in this chapter. The analysis of the optimal learning problem when the distribution of offers $(f(w))$ is unknown has previously been addressed in the literature by Rothschild (1974), Kohn and Shavell (1974), Axell (1974) and Rosenfield and Shapiro (1981). As noted in chapter 2, the results of this literature are not particularly useful in answering the above questions. Learning may 'destroy' much of our earlier analysis if it causes the optimal strategy to not possess the reservation

property (Rothschild (1974) and causes individuals with perfect recall to search longer (Kohn and Shavell (1974): Theorem 5).

In this chapter we consider a more restricted problem and look at the effects of learning when the only unknown is the probability of a successful search q . Since one interpretation of a model where offers are rationed is as a model of labour markets where wages fail to adjust allowing for uncertainty regarding quantities seems a reasonable approach. Considering only q to be unknown offers a considerable simplification over models incorporating more general imperfect information. In particular information enters into the individual's decision problem in a very discrete way. Either an offer is obtained or it isn't.

In the next section we again start by considering our simple vacancy search model of the last chapter. Within this framework we can examine how the idea of learning about q can be incorporated into a model of search and consider the effect of uncertainty about q in determining whether the individual will participate in the labour force or not. Many of the essential features of a more general search model where learning about q is allowed are visible even in this very simple case.

In section 3 we again consider a search model with wage dispersion and start by formulating a general learning problem and then presenting the model with learning about q as a special case. The extent to which the effects of learning can be analysed are discussed and some numerical results presented that suggest more fully than has previously been possible what the effects of learning in a job search context are.

The final section of this chapter summarises the results and makes some suggestions for future research in this field.

3.2. Learning in the Simple Vacancy Model

We start by taking the simplest formulation of the vacancy search model of the last chapter. Jobs once obtained are assumed to last for ever, unemployment benefits are ignored and following Lippman and McCall (1976a) we ignore discounting over the actual course of search and define the value of a job as $W = \frac{w}{(1-\rho)}$. Assuming an infinite search horizon the value functions in the case where q is known to the individual can be written

$$(3.1) \quad V_s = -c + qV_E + (1-q)V_s$$

$$(3.2) \quad V_E = W = \frac{w}{(1-\rho)}$$

Search will be worthwhile provided that $\frac{c}{q}$ is less than W , in other words, provided the expected gain of one more search (qW) is larger than the cost (c).

We now want to consider the effect of uncertainty regarding the parameter q . It is natural to incorporate such uncertainty by positing some probability distribution over possible values of q so that

$$(3.3) \quad q \sim h(q, \theta)$$

h is the probability density function (pdf) of q parameterised by some vector θ . h is defined over the interval $[0,1]$.

If the function $h(q, \theta)$ represents the individual's immutable

beliefs regarding the parameter q , then incorporating uncertainty will not affect decision making in this context. The maximand of the individual is expected lifetime income so that our individual is risk neutral. All we need to do is to replace q with

$\int_0^1 h(q, \underline{\theta}) q dq (=E(q))$ in (3.1) and for cases where the individual's beliefs are unbiased ($E(q) = q$) exactly the same condition for the value of search will result.

Any consequence of uncertainty regarding q that is to arise in a model such as this one where risk neutrality is assumed must arise because beliefs are not held with certainty and the individual learns over the course of search. We shall analyse the problem when information acquired over the course of search is incorporated into the individual's (subjective) prior pdf $h(q, \underline{\theta})$ in an optimal Bayesian fashion. The probability density that an individual holds given an observation (the posterior) must be the prior density multiplied by the probability (according to the prior) of the observation.

If $h(q, \underline{\theta})$ forms a parametric family of distributions for the unknown q , then all updating of the individual's beliefs arises through changes in the parameter vector $\underline{\theta}$.

The complete specification of the learning problem in our simple vacancy search model therefore requires knowledge of the functional form $h(q, \underline{\theta})$.

Each 'search' in the context of the above problem constitutes a Bernoulli trial with parameter q . Success (an offer) occurs with probability q and failure (a null) with probability $(1-q)$. The problem of estimating the probability q given a sequence of

observations is a common one in statistical decision theory and has its origins in such problems as determining by experiment the 'fairness' of a coin. The Beta distribution turns out to be appropriate in this case.¹ The Beta distribution is parameterised by two parameters which we will denote y, z and can be written as

$$(3.4) \quad h(q, \theta) = \frac{\Gamma(y+z)}{\Gamma(y)\Gamma(z)} q^{(y-1)} (1-q)^{(z-1)}$$

Γ is the gamma function and is given by

$$(3.5) \quad \Gamma(y) = \int_0^{\infty} u^{(y-1)} e^{-u} du$$

The expectation of a Beta random variable is given by $\frac{y}{(y+z)}$ whilst the variance is $\frac{yz}{(y+z+1)(y+z)^2}$.

In order to complete our specification of the problem we need to know how sample information is included into the parameters y and z . Given a prior distribution that is Beta with parameters y, z application of Bayes theorem dictates that the observation of an offer results in a posterior distribution (also Beta) with parameters $y+1, z$. The observation of a null offer results in a posterior distribution with parameters $y, z+1$.

Total ignorance regarding q is most sensibly allowed for by assuming a prior distribution that is Uniform on the interval $(0,1)$

1. See for example DeGroot (1970).

and again the Uniform (0,1) distribution is a member of the Beta family with parameters 1,1.

The individual's information set can therefore be seen as a set of observations on the success or otherwise of search. In the simple vacancy search setting of course, as soon as an offer is realised it is accepted, but one can imagine perhaps individuals starting the process with exogenous information, either of the observed search experience of others or of earlier spells of search unemployment.

We can now write down the value of search for an individual who currently has information represented by y, z and who will update his/her beliefs in the light of search experience. Rewriting (3.1) we have

$$(3.6) \quad V_s(y, z) = -c + \int_0^1 h(q, y, z) q dq \cdot V_E + \left[1 - \int_0^1 h(q, y, z) q dq \right] V_s(y, z+1)$$

The integral term is simply the expectation (conditioned on y, z) of q which we shall henceforth denote $q^E(y, z)$.

The value of search now depends not only on the wage offer, expectation of q and cost of search, but also upon the value of search conditioned on the observation of a 'null' offer. We cannot therefore solve (3.6) directly.

We can, however, find an intuitive argument as for why the ability to learn might make participation less likely. Consider two individuals i and j each of whom holds the same expectation

of q but one of whom, j , holds this expectation with certainty. Furthermore, assume that i will learn from only one observation so that having observed a null offer, i believes with certainty that $q = y/(y + z + 1)$. In that case

$$(3.7) \quad v_s^1(y, z) < v_s^0(y, z)$$

where superscripts denote the number of searches when learning will take place. (3.7) states that an individual who is prepared to learn from only one piece of information regarding q has a lower value of search than an individual who will never learn. By simple recursion it follows that

$$(3.8) \quad v_s^0(y, z) > v_s^1(y, z) > v_s^2(y, z) \dots > v_s^n(y, z).$$

Notice that the variance of the individuals estimate of q is for finite y, z arbitrarily close to zero and decreasing in y and z . Hence complete or almost complete learning is ensured in finite time and thus (3.8) shows our result, since $v_s^0(y, z)$ is individual j 's valuation function whilst $v_s^n(y, z)$ is individual i 's valuation.

Following the work of Kohn and Shavell (1974) it is usual to think of learning as increasing search. As noted earlier such a result depends crucially upon the perfect recall assumption in a wage search model. Here in a simple model of vacancy search learning turns out to be a bad thing (discourages search) for the individual because all news is bad news. The intuition underlying Kohn and Shavell's (1974) result is that learning can result in good or bad information but that recall ensures that bad news is effectively

insured against. In the present case all that the individual can 'learn' is that q is smaller than (s)he first thought (in expectation), therefore the expected value of search where learning is possible is smaller than in its absence.

The above result does not say anything about the degree of uncertainty with which individuals hold their views. There is an important distinction to be made here between a little 'learning' and none at all (for which the result holds) and a little 'learning' and a little less (for which we have no result as yet). What we are considering in the second case is the diffuseness of a searcher's prior distribution and how it affects search behaviour. Casual inspection of the form of (3.6) suggests such a result will be very difficult to obtain because we wish to compare two individuals who have different current values of both y and z . This is because to find a result on diffuseness we must hold the expected value of q fixed. There is therefore no way in which the two individuals could hold the same views given some search experience (only z is updated over the course of search).

We can, however, employ a rather different recursion argument that suggests rather than proves a result. Again index the two individuals by i and j and let both have the same expectation of q , but allow i to hold these views with less certainty (the variance of $h(q,y,z)$ is greater for i).

Taking a point where both individuals are nearly certain, it follows that the value of further search for i is less than that for j . Both individuals can only acquire bad news but i can only acquire more of it. Therefore we have the following

$$(3.9) \quad v_{si}^0(\cdot) < v_{sj}^0(\cdot)$$

Again superscripts denote the fact that we are considering what might be termed the learning horizon. We shall assume that this horizon is reached by both individuals approximately together.

Next notice that with one search to go before certainty is reached that

$$(3.10) \quad \left| v_{si}^1(\cdot) - v_{si}^0(\cdot) \right| < \left| v_{sj}^1(\cdot) - v_{sj}^0(\cdot) \right|$$

so that it follows that

$$(3.11) \quad v_{si}^n(\cdot) < v_{sj}^n(\cdot)$$

which establishes our result that i has a lower value of search (higher w^*) than j . Notice that the assumption of equal certainty horizons was needed in establishing this result. This requires that the variance of i 's subjective distribution on q should decline more quickly with observations than that of j , since i starts with a higher variance.

In fact considering the formula for the variance of a Beta distribution reveals that it is indeed the case that

$$(3.12) \quad y_i > y_j, z_i > z_j \Rightarrow \partial \text{var}_i / \partial z_i > \partial \text{var}_j / \partial z_j$$

(3.12) states that the larger are both parameters the more quickly does the variance decrease with z .

Indeed, for many cases (particularly for small expectations of q) the initially more uncertain individual will become 'certain' sooner.²

2. As an example of this phenomena consider x, y of (1,1) or (2,2) (expectation = $\frac{1}{2}$), and notice that the variance after 6 null offers is respectively 0.012 and 0.016. (Initial variance is 0.08 and 0.05 respectively.)

This seems a rather counter intuitive result and indeed it is a particular feature of the Beta distribution and the fact that information enters in a discrete fashion. It does, however, suggest that our intuitive result concerning the effect of diffuseness on the participation decision is correct in this case.

The result says that of two searchers with the same prior expectation, but one of whom holds a more diffuse view of the world, it is the latter who will be less inclined to search (or conversely more inclined to discouragement).

This kind of result is particularly interesting because it concerns risk neutral individuals. It suggests that expectations may be 'rational' (i.e. correspond to the true parameter q in the labour market) individuals risk neutral and yet an increase in subjective uncertainty will result in a real effect, namely a decrease in participation. Our earlier result suggested that 'learning' itself might also decrease participation.

The intuition behind our second result is clear, greater uncertainty leads to the conclusion that more bad news is possible, furthermore the form of the Beta distribution suggests that the unfavourable state of the world will be viewed as more certain more quickly if the individual has initially weak views.

In this section we have analysed learning in the context of the simple vacancy search model introduced in chapter 2. Up till now the literature on 'adaptive' search as the learning case is often called, has suggested few useful results. The great difficulty involved in solving for the optimal search strategy (even numerically) has meant that the consequences of learning within a search context have remained relatively unexplored.

Here, by taking a particular form of the problem, we have been able to get somewhat further and examine the issue of learning as it relates to the participation decision when the probability of receiving an offer (of known value) is unknown.

The main conclusions are that a) learning decreases the value of search in this context and therefore makes search less likely, b) having more to learn *ceteris paribus* is a bad thing and suggests again that search is less likely. The strength of these results stems from the fact that in the vacancy search model all 'news' is bad news, as soon as an offer is received it will be taken up and further learning ceases.

Whilst only the simplest formulation of the vacancy search model was considered here a number of generalisations are easily possible. Discounting over the course of search will not in any way affect our conclusions. We need only rewrite (3.6) as

$$(3.13) \quad V_s(y, z) = -c + \rho [q^E(y, z)V_E + (1 - q^E(y, z))V_s(y, z+1)]$$

Discounting of course introduces a time cost to search and therefore reduces the value of search compared with the next best alternative (permanent unemployment).

Incorporating a risk of unemployment and therefore of repeated spells of unemployment could also easily be accomplished. In the context of a learning model this possibility has somewhat more interesting implications. It is presumably those individuals with most spells of unemployment who have the best information about the search environment (q). The simple model therefore predicts a

relationship between labour market experience and the participation decisions. Of course past information may be a 'bygone' either forgotten or ignored because it is thought to be out of date or irrelevant.

A finite planning horizon could also be allowed for although for very short horizons the proof of our results (which depended upon full information being a possibility) would not necessarily go through.

Whereas in the perfect information case the infinite horizon assumption ensured a stationary environment so that nothing changed over the course of search in the learning case, information acquired by the individual may well cause a change of mind. The observation of one more 'null' may be sufficient to convince the individual that search is no longer worthwhile.

The learning model is therefore a considerably richer vehicle of analysis than the simple static vacancy search model of the last chapter. It allows for an examination of the role of 'uncertainty' which is seen to matter even to risk neutral searchers. It allows for the possibility of past information and experience influencing the participation decision. Finally, it allows (even with a stationary environment) an element of dynamics to enter into participation decisions.

In the next section we generalise the learning model reviewed here to incorporate the possibility of wage dispersion. One way of viewing the null offer model with learning is as a special case of learning about the distribution of wages. As a special case analysis becomes somewhat easier than that in the literature to date.

Simple generalisations of our results above do not, however, follow. A decision to reject some offers made means that over the course of search both good and bad information can be accumulated. Optimal search decisions and the value of search when learning is allowed thus depend upon the balancing of good and bad 'news'. We examine two cases, one where full insurance against bad news is provided by the perfect recall facility, one where no such insurance is possible. In the first case the existing literature suggests a rather weak result; in the second case no result at all. We therefore use a numerical simulation to suggest under what circumstances the analysis of this section might generalise.

3.3. Learning about Null Offers with Wage Dispersion

A model with wage dispersion allows us to consider the effect learning about q has on the decision of how long (in expectation) to search. Considering wage dispersion also involves us in the analytic difficulties associated with job search; in such a setting it turns out that the results demonstrated in the last section do not necessarily hold for this more difficult case.

We start by reformulating the wage dispersion model where null offers are allowed (considered in chapter 2) to allow for learning about null offers. Since the assumption regarding an individual's ability to revisit offers turns out to be crucial in the case of a model with learning we also generalise our analysis to consider the perfect recall world (where offers once made are never withdrawn).

We start by noting that q is a parameter in the 'full' wage distribution $g(w)$ facing the individual. Again using for notational

convenience W for $\frac{w}{(1-p)}$ we can write down the learning analogue of equation (2.9) which, for the case of search in the absence of recall (within search discounting is again ignored and b set to zero) can be written

$$(3.14) \quad V_s(\bar{W}, \underline{\theta}) = \max \left\{ \bar{W}, -c + \int_0^1 \int_0^\infty V_s(W, \theta(W)) g(W|q) h(q, \underline{\theta}) dW dq \right\}$$

$\theta(W)$ indicates the parameter vector $\underline{\theta}$ updated in the light of an offer W .

Equation (3.14) is in a form that can be equated with Lippman and McCall (1976a) p. 174. The fact that $g(w)$ is conditioned on q is explicitly modelled.

Rothschild (1974) shows that the individual's search strategy has a reservation form only in the ex post sense. Provided $\underline{\theta}$ contains information on \bar{W} then the reservation property is ensured.³ We shall consider only this case.

In the present case we are able to considerably simplify the general form (3.14) by noting that;

- a) the only distinction between an offer from an information point of view is whether it is a 'Null' or positive
- b) q simply premultiplies $V_s(\)$ and therefore may be integrated out.

Using these facts we obtain;

3. A priori the individual may decide to accept a null (quit search) if the next offer is zero but reject some positive wage offers because the observation of any positive offer informs him that search is favourable. Hence the acceptance set is discontinuous.

$$(3.15) \quad v_s(\bar{w}, \underline{\theta}) = \max \{ \bar{w}, -c + q^E(\underline{\theta}) \int_0^\infty v_s(w, \underline{\theta}(1)) f(w) dw \\ + (1-q^E(\underline{\theta})) v_s(0, \underline{\theta}(0)) \}$$

where $\underline{\theta}(1) = \underline{\theta}$ updated in the light of an offer and

$\underline{\theta}(0) = \underline{\theta}$ updated in the light of a null

We can also rewrite (3.15) explicitly in terms of the parameters of the Beta distribution for q .

$$(3.15a) \quad v_s(\bar{w}, y, z) = \max \{ \bar{w}, -c + q^E(y, z) \int_0^\infty v_s(w, y+1, z) f(w) dw \\ + (1-q^E(y, z)) v_s(0, y, z+1) \}$$

The value of search is defined 'recursively' and depends upon the value of search given two alternative outcomes an offer or a null.

So far we have only considered search where previous offers cannot be returned to. Whilst this seems a plausible assumption in a labour market with limited vacancies where vacancies may be filled, it is useful to consider an alternative extreme assumption, namely the case where any offer may be returned to so-called perfect recall.⁴

If we use \bar{w}_m to denote the maximum wage offer received to date, we can write the value of search as:

4. It is important to distinguish between recall and memory. It is assumed that individuals always remember (in particular regard to the information content) offers received, but may not be able to 'recall' them.

$$(3.16) \quad V_s(\bar{w}_m, y, z) = \max \{ \bar{w}_m, -c + q^E(y, z) [F(\bar{w}_m) V_s(\bar{w}_m, y+1, z) + \int_{\bar{w}_m}^{\infty} V_s(w, y+1, z) f(w) dw] + (1-q^E(y, z)) V_s(\bar{w}_m, y, z+1) \}$$

Here we have to take into account the fact that any positive offer received less than the current maximum suggests a value of search simply increased by the suggested change in information. Any offer received acts as a form of insurance against subsequently learning that offers are very scarce.

Provided that a reservation strategy is optimal which can be shown (see Kohn and Shavell (1974)), we can write a recursive form for the reservation wage corresponding to equations (3.15) and (3.16). Remember that intuitively the reservation wage is that wage which, if offered, equates the cost and benefit of one more search. In the case of no recall r satisfies

$$(3.17) \quad r(y, z) = -c + q^E(y, z) E \max (w, r(y+1, z)) + (1-q^E(y, z)) \max (0, r(y, z+1))$$

Whilst in the case of (3.16) we have

$$(3.18) \quad r(y, z) = -c + q^E(y, z) [r(y+1, z) F(r(y+1, z)) + \int_{r(y, z)}^{\infty} w f(w) dw] + (1-q^E(y, z)) \max (r(y, z), r(y, z+1))$$

Since both parameters y and z now influence the reservation wage and both can vary over the course of search, it is impossible to derive much in the way of analytic results regarding the effects of learning.

The result due to Kohn and Shavell (1974) mentioned previously applies to (3.18). Comparing (3.18) with the solution to the reservation wage in the case of perfect recall and no learning which can be written

$$(3.19) \quad r = -c + q[rF(r) + \int_r^{\infty} Wf(W)dW] + (1-q)r$$

Setting q in (3.19) equal to $q^E(y,z)$ in (3.18), it can be seen that the observation of a null offer is insured against since $r(y,z)$ is the minimum value that further search can attain. Given that the last term in (3.18) cannot be less than that in (3.19) and given that the second term is at least as great, it follows that the reservation wage given learning is greater in the recall case.

Nothing can be said regarding the effect of increasing subjective uncertainty, however, since this involves the comparison of two 'learning' reservation wages and no simple recursive arguments used earlier can be applied here (where both parameters change). We can however consider two individuals, i and j , who will both 'learn' from only one search and then become certain. Where i is the individual with greater subjective uncertainty (higher variance of q) i will have the higher reservation wage. This follows directly from the fact that according to i 's beliefs, the expectation of q given an offer will be greater than j 's. Since in the event of null the insurance effect of recall ensures that i

values further search equally to j the above result holds.

In the case of no recall we can make no statements at all about the consequences of learning. This is particularly disturbing since the no recall case is far more appealing in the context of a labour market with offer rationing. We can attempt to analyse perhaps a simplified version of the adaptive no recall search model in an attempt to generate some results that can subsequently be tested for robustness using numerical analysis. One way to simplify analysis is to consider a very short (2 period) horizon problem and see what this suggests about the effects of learning in the absence of recall.

Assume that only two searches are possible and denote the reservation wages when two searches remain as r_1, r_0 . We can solve this sort of problem recursively starting with period 1.

Firstly it is obvious that,

$$(3.20) \quad V_{s0}(W, y, z) = \bar{W} \quad \forall y, z$$

After the last search has been made the individual must take whatever is observed on the last search. The reservation wage at time 0 is zero.

Using the recursive formulation we have also that

$$(3.21) \quad r_1(y, z) = -c + E \max (r_0(y+1, z)W)q^E(y, z) \\ + (1-q^E(y, z)) \max (0, r_0(y, z+1)).$$

From the above, however, $r_0(y,z) = 0 \quad \forall y,z$ hence

$$(3.21a) \quad r_1(y,z) = q^E(y,z)E(W) - c$$

Only the expected value of q matters with one search to go since further learning will have no effect (the next offer will be accepted whatever it is).

When two searches remain we therefore have

$$(3.22) \quad r_2(y,z) = -c + q^E(y,z) \int_0^{\infty} \max(W, r_1(y+1,z)) f(W) dW \\ + (1-q^E(y,z)) \max(0, r_1(y,z+1))$$

Provided that search next period is worthwhile, we can rewrite the above (using the earlier formula for $E \max [W, r]$) as

$$(3.22a) \quad r_2(y,z) = -c + q^E(y,z) [r_1^+ + \int_{r_1^+}^{\infty} (W - r_1^+) f(w) dw] + (1-q^E(y,z)) r_1^-$$

Where $r_1^+ = q^E(y+1,z)E(W) - c$

and $r_1^- = q^E(y,z+1)E(W) - c$

Within this simple two period case the effect the increasing diffuseness of prior beliefs can be seen as an increase in $q^E(y+1,z)$ and a decrease in $q^E(y,z+1)$. It is important to note that in general these two effects will not be symmetric. By assuming that they are, we are concentrating on only 'small' increases in diffuseness.

Given that the two effects above are roughly the same, the net effect on the reservation wage (r_2) depends upon

$$(3.23) \quad \left| \frac{\partial r_2(y,z)}{\partial q^E(y+1,z)} - \frac{\partial r_2(y,z)}{\partial q^E(y,z+1)} \right| \quad \text{sign}$$

Applying the chain rule we have

$$(3.23a) \quad \frac{\partial r_2(y,z)}{\partial q^E(y+1,z)} = \frac{\partial r_2}{\partial r_1^+} \cdot \frac{\partial r_1^+}{\partial q^E(y+1,z)}$$

$$(3.23b) \quad \frac{\partial r_2(y,z)}{\partial q^E(y+1,z)} = \frac{\partial r_2}{\partial r_1^-} \cdot \frac{\partial r_1^-}{\partial q^E(y+1,z)}$$

And since we are considering only a two period problem

$$\frac{\partial r_1^-}{\partial q^E} = \frac{\partial r_1^+}{\partial q^E} = E(W).$$

We can therefore rewrite (3.23) as

$$(3.24) \quad Z = E(W)(2q^E(y,z) - 1) - E(W)(1 - F(r_1^+))$$

where if $Z > 0$ increasing diffuseness increases the reservation wage and vice versa.

It follows immediately from (3.24) that if $q^E(y,z) < 0.5$, r decreases with greater uncertainty. For $q^E > 0.5$ the effect of

increasing diffuseness is ambiguous and depends upon the cost of search. For some small values of search cost there exists a possibility that increasing subjective uncertainty increases r .

The simple two period model suggests that when recall is not possible risk neutral searchers will search less the greater is their subjective uncertainty. This is again an interesting result along the lines of the result for the vacancy search model. In order to see whether it continues to hold when the search horizon is not fixed arbitrarily at two periods, we must numerically evaluate optimal reservation wages for a model of search with learning about null offers.

What the discussion so far has suggested is the considerable analytic difficulty encountered when dealing with search models incorporating learning. When wage dispersion is present we have not been able to establish the effect of subjective uncertainty in any general way either for search with or without recall. In the case of recall we have a result that 'learning' is better (results in a larger expected gain from search) than static search whilst in the no recall case we have a result that suggests the opposite.

Within our restricted model we do have a straightforward recursive expression for the (information contingent) reservation wage, which we can employ in a numerical simulation. We can therefore use numerical results to examine the unanswered questions and check the generality of our analytic results.

The algorithm used and Fortran programming details of numerical simulation in this context are detailed in the Appendix of this thesis, we will therefore not discuss them further here. As a benchmark

case we start with numerical solutions to equation (2.13)
that is, the static search reservation wage.

These results are all based on a truncated Normal wage offer distribution $\mu = 1,000$ $\sigma = 100$. The controlling parameter used was the cost of search and this was varied between values of 1 to 100. This range of costs of search gives rise to reservation wages that imply almost immediate acceptance ($c = 100$) to extremely lengthy search ($c = 1$). In table 1 we report r as a function of q given 5 values for c . These relationships are graphed in Figure 1.

TABLE 1

$c \backslash q$.1	.2	.33	.5	.66	.8	.9
1 r	1085	1108	1141	1154	1175	1181	1184
1 p_{ue}	.02	.03	.03	.03	.03	.03	.03
10 r	880	980	1020	1049	1065	1077	1084
10 p_{ue}	.08	.12	.14	.16	.17	.17	.18
50 r	502	750	780	909	941	961	972
50 p_{ue}	.10	.20	.31	.41	.48	.52	.55
100 r	0	502	697	800	852	881	896
100 p_{ue}	.10	.20	.33	.49	.61	.71	.76

Figure 1. Approximate graph of r as a function of q .

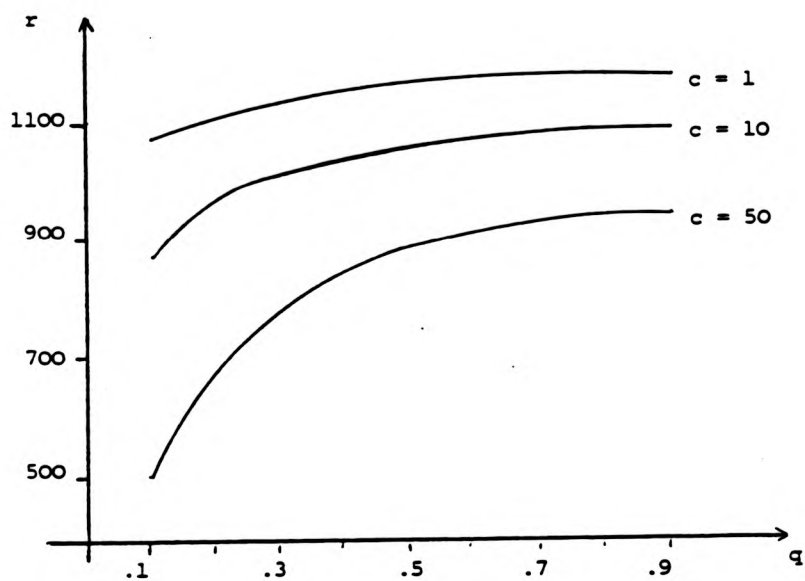


TABLE 3

RECALL

E(q)	Variance (q) $\mu = 1000, \sigma = 100, c = 10.$						
0.10	937. 0.07	918. 0.08	915. 0.08	917. 0.08	913. 0.08	910. 0.08	914. 0.08
0.20	1014. 0.09	1003. 0.10	997. 0.10	993. 0.11	991. 0.11	989. 0.11	988. 0.11
0.33	1051. 0.10	1042. 0.11	1037. 0.12	1035. 0.12	1031. 0.12	1031. 0.12	1027. 0.13
0.50	1069. 0.12	1062. 0.13	1058. 0.14	1057. 0.14	1055. 0.15	1054. 0.15	1052. 0.15
0.66	1077. 0.15	1074. 0.15	1072. 0.16	1072. 0.16	1071. 0.16	1070. 0.16	1070. 0.16
0.80	1081. 0.17	1080. 0.17	1078. 0.17	1078. 0.17	1077. 0.18	1077. 0.18	1077. 0.18
0.90	1084. 0.18	1084. 0.18	1084. 0.18	1084. 0.18	1084. 0.18	1084. 0.18	1084. 0.18

TABLE 4

RECALL

E(q)	Variance (q) $\mu = 1000, \sigma = 100, c = 100.$						
0.10	0. 0.10	0. 0.10	0. 0.10	0. 0.10	0. 0.10	0. 0.10	0. 0.10
0.20	490. 0.20	490. 0.20	490. 0.20	490. 0.20	490. 0.20	490. 0.20	490. 0.20
0.33	701. 0.33	702. 0.33	701. 0.33	701. 0.33	701. 0.33	701. 0.33	701. 0.33
0.50	804. 0.49	802. 0.49	801. 0.49	801. 0.49	801. 0.49	801. 0.49	801. 0.49
0.66	855. 0.61	854. 0.61	854. 0.61	853. 0.61	853. 0.61	853. 0.61	853. 0.61
0.88	881. 0.71	881. 0.71	881. 0.71	881. 0.71	881. 0.71	881. 0.71	881. 0.71
0.90	897. 0.76	897. 0.76	896. 0.77	896. 0.77	896. 0.76	896. 0.76	896. 0.76

These serve to show that the suggested analytic results are particularly robust with respect to the effect of increasing subjective uncertainty. Greater subjective uncertainty (a more diffuse prior) results in a larger reservation wage and therefore more lengthy search. As the costs of search increase so the gains from search fall and in particular the gains from information, so that subjective uncertainty has virtually no effect for high costs of search. In terms of gains from information the individual stands to gain most if he has a low initial expectation about q . Here the increasing effect of diffuseness is greatest because there is a chance that information of considerable advantage will be learnt over the course of search. For very small values of $q^E(y,z)$ there exists a possibility that the reservation wage is actually decreasing for some increases in diffusion, but this effect does not seem significant and in any case we know from our analysis that the value of search with learning (r) always exceeds that when no learning is possible.

These results then confirm and extend the analysis of the learning model when recall is allowed. With insurance against bad 'news' learning increases the expected value of search which is also generally increasing in the amount of learning that is possible. The value of learning will be greatest when initial beliefs suggest that a lot of good news may be possible. Finally, costs of search dictate the net benefit of information (it is of little value to know that offers are available if to obtain them is very expensive) and hence the effect of learning is greatest the lower are search costs.

In tables 5, 6 and 7 and figures 2 and 3 we present the results of a numerical simulation involving the learning model assuming no recall. These results are perhaps the most interesting because our analysis was able to say very little about the effects of learning and yet the no recall assumption is thought to be more realistic.

TABLE 5

NO RECALL

E(q)	+ Variance (q)						
	$\mu = 1000, \sigma = 100, c = 1$						
0.10	1042. 0.03	1067. 0.03	1074. 0.02	1078. 0.02	1080. 0.02	1081. 0.02	1083. 0.02
0.20	1088. 0.04	1108. 0.03	1114. 0.03	1116. 0.02	1118. 0.02	1119. 0.02	1119. 0.02
0.33	1120. 0.04	1136. 0.03	1141. 0.03	1143. 0.03	1144. 0.02	1145. 0.02	1145. 0.02
0.50	1146. 0.04	1161. 0.03	1163. 0.03	1164. 0.03	1164. 0.03	1165. 0.02	1165. 0.02
0.66	1172. 0.03	1175. 0.03	1176. 0.03	1177. 0.03	1177. 0.03	1177. 0.03	1177. 0.03
0.80	1183. 0.03	1184. 0.03	1184. 0.03	1185. 0.03	1185. 0.03	1185. 0.03	1185. 0.03
0.90	1189. 0.03	1189. 0.03	1189. 0.03	1189. 0.03	1188. 0.03	1188. 0.03	1188. 0.03

TABLE 6

NO RECALL

E(q)	+ Variance (q)				$\mu = 1000, \sigma = 100, c = 10$		
0.10	692. 0.10	832. 0.10	864. 0.09	878. 0.09	886. 0.09	890. 0.09	894. 0.09
0.20	843. 0.19	935. 0.15	954. 0.14	962. 0.13	966. 0.13	969. 0.12	971. 0.12
0.33	932. 0.25	992. 0.18	1004. 0.16	1009. 0.15	1012. 0.15	1013. 0.15	1015. 0.15
0.50	993. 0.26	1038. 0.18	1043. 0.17	1045. 0.16	1046. 0.16	1046. 0.16	1047. 0.16
0.66	1056. 0.19	1062. 0.18	1064. 0.17	1065. 0.17	1065. 0.17	1065. 0.17	1066. 0.17
0.88	1075. 0.18	1076. 0.18	1077. 0.18	1077. 0.18	1077. 0.18	1077. 0.18	1077. 0.18
0.90	1084. 0.18	1084. 0.18	1084. 0.18	1084. 0.18	1084. 0.18	1084. 0.18	1084. 0.18

TABLE 7

NO RECALL

(E(q))	+ Variance (q)				$\mu = 1000, \sigma = 100, c = 100$		
0.10	0. 0.10	0. 0.10	0. 0.10	0. 0.10	0. 0.10	0. 0.10	0. 0.10
0.20	119. 0.20	285. 0.20	339. 0.20	375. 0.20	399. 0.20	416. 0.20	428. 0.20
0.33	433. 0.33	551. 0.33	606. 0.33	634. 0.33	650. 0.33	660. 0.33	667. 0.33
0.50	617. 0.50	752. 0.50	776. 0.49	784. 0.49	788. 0.49	791. 0.49	792. 0.49
0.66	807. 0.64	836. 0.63	843. 0.62	846. 0.62	847. 0.62	848. 0.62	849. 0.62
0.80	872. 0.72	877. 0.71	878. 0.71	879. 0.71	879. 0.71	879. 0.71	880. 0.71
0.90	895. 0.77	896. 0.77	896. 0.77	896. 0.77	896. 0.76	896. 0.76	896. 0.76

First, it can immediately be noted from the tables that our suggestion derived on the basis of the two period model generalises for the infinite horizon case and is remarkably robust. Increasing uncertainty (in complete contrast to the recall model) actually reduces the reservation wage in the vast majority of cases. The possibility of bad 'news' dominates the individual's evaluation of the gains from search activity. Thus in this case, as with the simple vacancy search model, increasing uncertainty reduces search even for risk neutral searchers. This result is particularly robust and holds for all reported values in the tables. We were able to generate the opposite result by considering very small costs of search ($\approx .15$) but such values are both implausibly small and only caused a reversal for $q^E(y,z) > .8$. As in the case of search with recall the effect of uncertainty is greatest when the potential gains and losses are largest so that when q^E is small subjective uncertainty has very large effects.

As has already been noted increasing costs of search in the absence of recall suggest a larger effect - the individual is concerned with the expected cost of waiting which is greater the larger is c .

These results therefore suggest a natural extension to the results for our simple vacancy search model given learning about offer availability. The greater is subjective uncertainty, the less likely is participation in the labour market and given participation the less search is conducted. A model with wage dispersion enables us to conclude that if recall is not possible, learning is a bad thing (decreases the value of search) for even a risk neutral

individual because the possibility of learning information to the detriment dominates. This is not a result that has been available before and runs counter to the suggestions of analysis made with perfect recall as a maintained assumption. The models of this section generalise very easily along the lines suggested for the simple vacancy search model. The arguments used there will not therefore be repeated.

3.4. Summary and Applications

In this chapter we have considered a logical generalisation of the model of search incorporating offer constraint reviewed in chapter 2. Allowance was made for the fact that individuals may be imperfectly informed regarding the parameter q and learn about q over the course of search.

We started by considering a simple vacancy search model which enables an examination of the search participation decision. We were able to generate strong results within the context of this simple model regarding the role of imperfect information and learning. Uncertainty regarding q was seen as being a discouragement to search, greater subjective uncertainty led even risk neutral individuals to participate less. Even within this simple model, however, we needed to exploit the particular functional form of prior beliefs in order to generate unambiguous results.

We next considered the extension of the vacancy model to allow for wage dispersion and saw that the results here were much more difficult to obtain and depended crucially upon the assumption made regarding the availability of previously sampled offers.

The assumption previously maintained in the literature that all offers remain available gives rise to a net gain from imperfect information for risk neutral searchers. The ability to learn suggests a greater expected wage with the possibility of bad news occurring effectively insured against by recall. More learning, however, could not be demonstrated as always being better than less. Under the assumption of no recall, no results exist, suggesting the consequences of learning. We considered a simple two period example and deduced that in this case a reversal of the result for recall was rather likely. Increasing subjective uncertainty might lead to less search, imperfect information/greater dispersion was a bad thing.⁵

These analytic results were confirmed by our numerical analysis in the case of a Normal distribution of offers. Furthermore, the circumstances under which imperfect information was most important were outlined. Where imperfect information is a good (recall case) search costs devalue it where it is a bad (no recall) search costs negate it.

Throughout this and the previous chapter we have attempted to suggest the applicability of individual search models. Of the two possible uses outlined in chapter 1 it should be clear that the additional complexity of learning models makes them an impractical input into market equilibrium constructions. Such models do, however, offer richer analysis of individual decision making. In this chapter we have identified an important role for learning in

5. Dispersion in beliefs about q must be distinguished from objective dispersion of wages ($f(w)$). The latter always leads to greater search for risk neutral individuals, the former as we have suggested does not.

determining both participation and expected duration decisions. A no learning model portrays participation as a once and for all decision in a stationary environment. When learning is allowed the individual must constantly review his position. This corresponds very much more closely to the notion of discouragement (a process) rather than non-participation (a state). In states of the world where individuals hold diffuse priors, single observations may be sufficient to discourage further search. Where information is good and priors 'tight' such discouragement may only come about very slowly. Even if individuals' beliefs are unbiased ($q^E(y,z)=q$) and risk neutrality assumed, there may be value in information so that discouragement does not prevail. Again we are considering only a partial equilibrium model and we must guard against making equilibrium type conclusions. Extending to the wage dispersion case our preferred results (no recall) suggest that imperfect information might lead to sub optimal search.

The central insight of this chapter has therefore concerned the role that information or the lack of it might play in determining an individual's labour market decisions. Simple comparisons with decisions made under certainty regarding 'q' are not possible. An individual's response has been shown to depend upon the value of certain economic parameters (the cost of search) and upon the possibility or otherwise of returning to previously offered employment. This latter facility it has been argued acts as a form of insurance against discovering something unfavourable and therefore promotes greater search activity. In the absence of recall imperfect information may discourage search even to the extent of leading an individual to abandon labour market activity altogether. What are the applications of this analysis?

One way in which government actions can according to the models of this chapter affect outcomes is therefore through 'beliefs'. An increased faith in a government's ability to stabilise economic activity would suggest according to the analysis of this chapter a real effect upon search decisions. It may well serve to increase both participation and search activity in the economy, which we might in turn expect to have some effect upon the employment decisions of firms and hence turn out to be self-fulfilling beliefs. This kind of role for governments is very much at the heart of Keynesian macro-economic analysis and therefore suggests one application of our results towards a macro explanation of unemployment. What in a sense is most revealing in our results in this context is the minimal assumptions which give rise to real phenomena. We have not assumed risk aversion nor have we assumed that an individual's beliefs are in any sense irrational and yet a move towards greater (perceived) certainty might generate an employment 'boom'.

Whilst the analysis of this chapter is suggestive of macro applications it has been essentially micro economic. At the micro level a number of conclusions can therefore be drawn. The first thing to note is that 'information' is in a sense an additional explanatory variable. If we have a set of observations of individuals search behaviour which we wish to explain then the information that those individuals have will be important. We cannot unambiguously say whether better informed individuals search more or less but can conjecture that the second is perhaps a plausible possibility in many circumstances. Secondly, the analysis we have considered gives rise even in otherwise 'stationary' circumstances to time dependent behaviour, so that as we noted above an individuals

'discouragement' is not so much an event as a process. We argued in chapter 2 that the discouraged worker phenomena was perhaps an under exploited aspect of the search literature - one to which a consideration of 'rationing' in search automatically led. The analysis of this chapter is therefore perhaps most important in providing a more satisfactory account of discouragement in a search environment.

Discouragement occurs because an individual's labour market experience leads them to the conclusion that the import of more resources into the job seeking process is not justified. Is this a good or a bad thing? In as much as an individual's experience represents the true nature of the economic environment (s)he faces we might regard it as good. After all, the assumption we have made is that of individual rationality - an individual would not drop out unless to do so was 'for the best'. There does, however, exist the strong possibility that an individual's experience represents the luck of the draw. The best analogy here is to simple games of chance. Imagine being offered the chance to bet on the 'tails' outcome of a tossed coin not knowing its fairness. Three consecutive heads may well serve to persuade that the game is rigged and yet this 'unlucky' observation occurs 12½% of the time even with a fair coin. If individuals become discouraged from labour market participation by similar bad luck we may well perceive the need for corrective action even though given their information sets individuals are acting rationally. One response might well be the provision of information itself or the improvement of the mechanism whereby information is transmitted. The development of these ideas is left to future work.

Chapter 4

Job Search and the Firms' Decisions 1: Wage Determination

4.1. Introduction

In the previous two chapters we have been concerned exclusively with the decisions of job searching individuals and in particular upon the consequences of offer rationing for search decisions. In this and the next chapter we turn attention towards the decisions of employing firms in a search environment. In conventional terminology chapters 2 and 3 were concerned with the supply side of a labour market characterised by search, chapters 4 and 5 will concentrate on the demand side.

The conventional derivation of the demand for labour as contained in standard texts (Addison and Siebert (1980)) views a firm as taking wages as given and employing labour up to that point where the value product just equals the wage. Where labour is the only variable input, diminishing marginal productivity ensures that the employment decision is bounded and maximises the firm's profit. The assumptions that underlie this static profit maximisation approach ensure that time doesn't matter. In particular it is assumed that the labour input can be instantaneously and costlessly adjusted.

The introduction of worker search together with some notion of job turnover in the labour market is inconsistent with these last two assumptions. As we have seen in previous chapters it is a fundamental assumption of search models that search is costly and takes time. Furthermore, if job searchers chose amongst firms to contact randomly, any one particular firm will face an uncertain flow of job applicants. A second direct consequence of assuming search activity is that wage taking on the part of firms is no

longer plausible. To understand this it is necessary to understand that a firm once contacted by a job searcher is different to that searcher from other firms in the market. The reason for this is that to contact other firms the individual must engage in costly search. Again in conventional terminology every firm in a labour market characterised by search possesses monopsony power. Indeed the assumptions of perfect competition or wage/price taking are inappropriate in any market characterised by imperfect information, a point made as long ago as 1959 by Arrow.

The above comments suggest that there are two aspects of firms' decisions that we might consider when modelling the demand side of a 'search' labour market.

There is of course the conventional employment decision - although here the possible stochastic nature of applicants to the firm suggests that employment is a natural random variable not entirely the choice of a firm. We shall therefore replace employment decisions by vacancy decisions. If labour adjustment were instantaneous and costless these would be the same thing, where search is involved it is the latter that the firm determines in order to influence the former. From the point of view of our analysis in previous chapters it is vacancy decisions that give rise to the null offer phenomena. If firms accept applicants without limit, then all search is by definition successful. The point at which firms refuse applicants is that point at which null offers are received.

Secondly, there is the wage decision of firms, how much to offer an applicant. Again in a search setting this offer may have an uncertain outcome. If the minimum wage acceptable to an individual were known by a firm, then clearly the reservation wage would be

offered. If, however, as seems likely, reservations differ over individuals, some individuals will reject certain wage offers. Again the offered wage is a variable that a firm might set in order to affect its employment and profit.

We address these two decisions in reverse order, considering wage decisions in this chapter and examining vacancy creation in the next. One of the objectives of this chapter is to lay the groundwork for the analysis of both chapters 5 and 6. The second objective is an analysis of wage decisions. Such an analysis is necessary for an understanding of the wider implications of search for market outcomes. We have seen in chapters 2 and 3 how wages act as signals to individuals concerning the value of search and participation. Here we consider the factors that might be important in generating such signals. This issue has not previously been directly addressed in the literature although the work of Pissarides (1979), McKenna (1980) and Eaton and Watts (1977) are related to the current analysis.

In the next section of this chapter we consider a stylised model of a firm's wage and vacancy decisions in a search context that serves to highlight the areas of difficulty. In the remainder of the chapter we concentrate on the simplest case of offer rationing, that of a single vacancy firm. Wage setting behaviour is then subjected to examination. We are interested in isolating those parameters which affect a firm's wage offer and in determining the direction of change. It turns out that even within a very simple setting, determining a firm's wage offer proves rather difficult. The problems arise when job searchers have an unknown reservation wage from some p.d.f.

In both this and the following chapter our general approach is the following. We model a firm whose employment evolves over time according to a stochastic process. The parameters of this stochastic process are in part determined by firstly the search environment and secondly the vacancy creation and wage decision of the firm, in a manner which needs to be made precise. The objective of the firm is expected profit maximisation. In a dynamic context this is generally taken to be the discounted sum (or integral) of a finite or infinite stream of profits. Here we shall consider only the average per period profit, an approach that will be justified in section (4.3). In all cases separations from the firm (losses of employment) are considered exogenously determined by job turnover in the labour market. Our analysis is limited for the most part to models where labour is assumed to produce equal increments of output irrespective of total employment. This assumption is made for analytic tractability and in cases where numerical analysis is employed (chapter 5) is shown not to be critical. In all cases we consider the employment process to evolve in continuous time for reasons that will become clear as analysis develops.

4.2. A General Framework of Analysis

Throughout the following, w will refer to a firm's wage offer (which will be assumed to be constant over time) and L will denote the total number of job slots or vacancies that a firm creates. L is the cut-off point of applicants accepted.

In the conventional static theory of the firm, output is generally taken to be some smooth continuous function of capital and labour inputs. Here we shall assume capital fixed and consider

labour input as synonymous with the number of individuals employed ('hours' or 'effort' variables are ignored). Consider a firm that at time t has a labour force l (integer). Given a number of stochastic influences operating in a labour market, it is reasonable to expect individuals to leave the firm. Leaving may take many forms, quitting in order to find a better job (possibly search for a better wage or other job characteristics), layoff, illness or retirement. Here we assume that 'learning' is exogenous and therefore that the latter explanations are dominant. The exogenous leaving of individuals from a firm will be modelled by assuming a probability $\mu\delta t$ that an individual leaves the firm in a time interval δt . For suitably small δt the probability of more than one individual leaving is of order $(\delta t)^2$ and approximately zero. The advantage of working in continuous time is now clear. In continuous time we can concentrate attention on employment processes that change by a single increment rather than more general 'Markov' processes.

Once unemployed, individuals are assumed to search for new employment. It is of no consequence whether search is discrete, each contact taking exactly 1 week, say, to achieve or continuous, given a distribution of searchers over time, it will still be the case that the firm will be contacted periodically by job searchers. We will denote the probability of a contact to the firm (again relative to an interval δt) by $\gamma\delta t$. Of course, if individuals differ as to their reservation wages there will be a probability that the firm's wage offer w will be rejected. If we denote by $g(r)$ the probability density function of job searchers' reservation wages, then the probability that a job searcher contacting the firm will accept an offer of \bar{w} is simply $G(\bar{w})$ where $G(w)$ is the cumulative

distribution function. We can therefore write the probability that the firm's employment increases by one unit of labour in time δt (call this $\lambda \delta t$) as,

$$(4.1) \quad \lambda \delta t = \gamma G(w) \delta t$$

In the simple framework just described, the firm's employment level is a random variable which evolves through time. Increases in employment have been made dependent (through $G(w)$) on the firm's wage offer whilst decreases in employment occur through exogenous influences. The structure described above is perfectly capable of handling endogenous quitting. If for example search 'on the job' were possible, then one element determining the rate at which employment declines would again be the firm's wage offer. By paying a high wage the firm would reduce the probability of employment losses. We will, however, remain with the framework described above which is consistent with 'intensive' search as described in the previous two chapters.

Starting from some employment level \hat{l} at time t , the possible employment outcomes at time $t + \delta t$ can be shown via a simple probability tree (Fig.1).

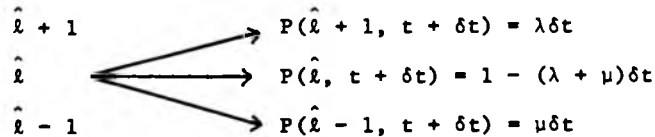


Figure 1

The probability of finding a firm at some employment level at a given time will therefore in general depend upon the initial state, the time elapsed since the initial state was occupied, the parameters γ , μ and the firm's wage offer w . We shall have

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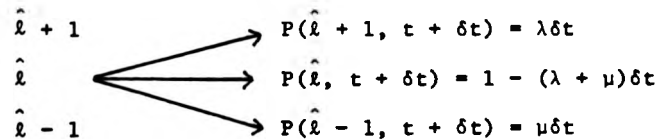


Figure 1

The probability of finding a firm at some employment level at a given time will therefore in general depend upon the initial state, the time elapsed since the initial state was occupied, the parameters γ , μ and the firm's wage offer w . We shall have

cause to simplify analysis by considering only steady states of the employment process, in which case only the last two factors enter into the problem.

So far attention has been confined to describing the environment in which a firm operates. It is now time to consider the value to a firm of being in a particular employment state, and of transmitting through states over time.

4.3. Production and Profit

In line with conventional theory of the firm, we assume here that firms produce output (Q) sold on a competitive market at price \bar{p} . To keep things as simple as possible, consider product market conditions fixed.

As far as factor inputs are concerned we shall consider capital fixed and consider output to vary with employment, so that we can write total revenue as

$$(4.2) \quad TR = \bar{p}Q(\ell) \quad ; \quad Q'(\ell) > 0, \quad Q''(\ell) \leq 0$$

where $Q(\ell)$ is a smooth continuous production function. The costs of producing a given output are in the form of a wage bill ($w\ell$) and rental on capital equipment. This capital rental we shall denote by a general cost function $k(L)$. The argument of this function L , denotes the total number of vacancies that a firm creates. In fact the assumptions embodied in this formulation are rather appealing. If one views 'machines' and 'men' as complements in production, then the function $k(L)$ represents the cost of a fixed number of machines.

In fact in this chapter we shall abstract from such capital costs and consider only firms facing a simple cost \bar{k} . However, we shall see in chapter 5 that reintroducing the cost of capital function naturally endogenises vacancy creation decisions.

In the meantime we can simply write the flow of profit to a firm in state l as

$$(4.3) \quad \pi = \bar{p}Q(l) - wl - \bar{k}$$

Of course the state that a firm occupies varies over the course of time according to a probability process so that the value of a stream of profits is presumably what motivates a firm's decisions. We can again use the 'principle of optimality' to write a dynamic programming value function for the firm.

$$(4.4) \quad V(l, t) = \pi(l, w)\delta t + \lambda\delta t V(l+1, t-\delta t) + \mu\delta t V(l-1, t-\delta t) \\ + (1-\lambda\delta t-\mu\delta t)V(l, t-\delta t)$$

In equation (4.4) time is to be thought of as running backwards, hence $V(l, t)$ is the maximum value of a firm's profit stream when time 't' remains and the current employment state is l . Discounting is being ignored here for simplicity, but can be incorporated quite simply.

To solve equation (4.4) we must firstly rearrange terms and take limits to yield a differential equation in $V(\cdot)$ and then solve a set of $(L+1)$ such equations simultaneously to yield an explicit functional form. Not surprisingly this approach is not feasible

except in cases of small L . We are interested in this chapter in examining the optimal wage decisions of firms when the number of vacancies is exogenously fixed.

If we ignore discounting but consider an infinite time horizon, total profits become unbounded, however 'average' per period profit will be (under certain conditions) a bounded and well behaved function of the firm's decision variable w .

Indeed, we can write an expression for expected per period profit directly

$$(4.5) \quad E[\pi] = \sum_{\ell=0}^L p(\ell) [\bar{p}Q(\ell) - w\ell - k]$$

where $p(\ell)$ has the interpretation of the long run or steady state probability of a firm occupying state ℓ . $p(\ell)$ may be thought of as the proportion of time that a firm spends in state ℓ in the 'long run'. Provided that $p(\ell)$ can be determined, equation (4.5) gives a direct expression for which maximising choices of the firm will be considered. It forms the basis of the analysis of this chapter and the next.

The key assumptions in making this simplification are the lack of discounting and the infinite horizon. We are therefore abstracting from all forms of impatience in modelling firms' decisions. As we shall see later, the effect of discounting can be demonstrated and conforms with intuition; it is considered more important in this chapter to have a general grasp of the determination of firms' decisions in a search context. Some loss of generality is therefore the price which must be paid.

One further simplification to analysis follows when the form of $Q(l)$ is linear (i.e. $Q(l) = \alpha l$). Then the expression for expected profit depends not on all the state probabilities separately, but rather on the expected steady state employment of the firm. Hence (4.5) becomes

$$(4.5a) \quad E(\pi) = (\beta - w)\bar{l} - \bar{k}$$

$$\text{where } \beta = \bar{p}\alpha \quad \bar{l} = \sum_{l=0}^L lp(l)$$

Where a particular closed form expression is possible for \bar{l} (4.5a) represents the most compact analytic form for considering a firm's wage decisions. Of course linearity of the production function is a strong assumption but does often provide a useful first case.

We have in this and the previous two sections described a framework of analysis. In the following two sections we consider the application of this framework to an analysis of a firm's wage decisions in a market characterised by search. We consider in turn two specialisations of the above model, both of which abstract from decisions regarding the creation of vacancies.

In the first of these specialisations the firm is assumed to possess only a single job slot so that the effect of search and job turnover is to create at most a single vacancy at the firm. This is the simplest model consistent with the generation of 'null' offers since those searchers who contact the firm when the single job slot is occupied will be unsuccessful. The determinants of a firm's wage offer to applicants for this single vacancy are examined.

To contrast with this 'null' offer case we then consider a firm that can expand employment without limit, that is to say that every searcher who contacts the firm will be made a wage offer, all search is successful. Again we examine the firm's wage offer in these circumstances under the assumption that a single wage is paid to all employees.

We will therefore have a model of monopsony wage determination in a search environment.

4.4. The Single Vacancy Firm

In this section we submit to formal analysis a firm described as in section 4.3. under the assumption that $L = 1$. There are two interpretations of this extreme limiting case of rationed vacancies. The first has already been alluded to above. One can think of this model as one of the short run decisions of a firm which has fixed capital stock and for which the possibilities of labour/capital substitution are highly limited. This is quite appealing. A second interpretation which also has appeal in a different context, considers the firm as being one facing a strong constraint on sales in its product market. The limiting case of course considers a firm that can sell the output of only a single employee. For such a constraint to bind implies that output is highly perishable.

The first interpretation seems appropriate to a market clearing approach to vacancy rationing, the second to a non market clearing approach.

Maintaining the idea that it is expected steady state profit that a firm maximises, we need to derive expressions for the probability

(in steady state) that a single vacancy firm has a vacancy (state 0) or is employing (state 1).

The method of deriving steady state probability distributions of stochastic processes is detailed in Cox and Miller (1970) and we will not be concerned with formal proofs here. The proportion of time that a firm spends in each state in the long run reflects the probabilities of transmitting between states and not the initial state. Hence we can write¹

$$(4.6) \quad \lambda p(0)\delta t = \mu p(1)\delta t$$

Furthermore the probabilities $p(0)$ and $p(1)$ must sum to unity so that (4.6) can be solved to yield

$$(4.7) \quad p(0) = \frac{\mu}{(\lambda+\mu)}, \quad p(1) = \frac{\lambda}{(\lambda+\mu)}$$

Remembering that μ is exogenous to the firm whilst λ depends through $G(w)$ on the firm's wage offer equations (4.7) show the expected dependency of 'state' on wage. $p(0)$ is decreasing whilst $p(1)$ is increasing in w . The higher is the wage offer the greater the probability of a randomly contacting job searcher accepting it and therefore the greater the proportion of time a firm spends with its vacancy filled.

Substituting (4.7) into (4.5a) and differentiating yields, a first order condition (foc) for expected profit maximisation of the form

1. Cox and Miller (1970) p. 356

$$(4.8) \quad \frac{\partial E(\pi)}{\partial w} = \frac{(\beta-w)\mu\lambda_w}{(\mu+\lambda)^2} - \frac{\lambda}{(\lambda+\mu)} = 0 \Rightarrow \frac{(\beta-w)\mu}{(\mu+\lambda)} = G(w)$$

(4.8) is of course necessary for a maximum, we also require that the derivative (wrt w) of (4.8) be negative for sufficiency.

As w is increased the profit from a filled vacancy falls whilst the proportion of time a vacancy is filled increases. Setting a wage so as to maximise profits entails balancing at the margin these two effects. Hence the first term of (4.8) is the increase in profit due to increased employment whilst the second term is the loss in profit due to a lower wage/value product margin.

The actual shape of the profit function will of course depend upon the distribution of reservation wages, but since $(\beta-w)$ equates to zero for some w , it will in general be a well behaved function of the wage offer. One possibility is drawn in Figure 1.

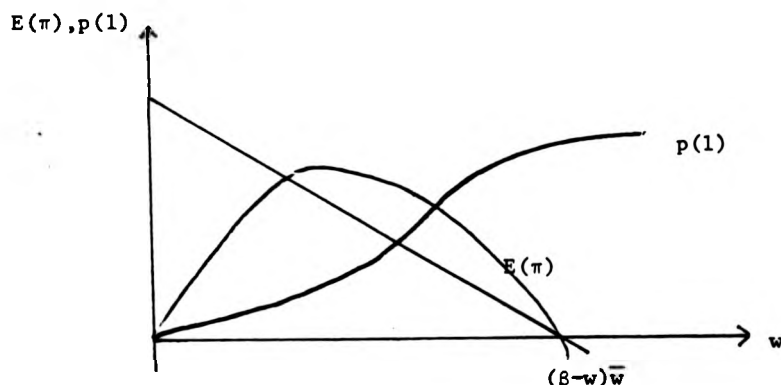


Figure 1. Expected Profit and w

$p(1)$ is monotonic in w and approaches $\frac{\gamma}{(\gamma+\mu)}$ as $w \rightarrow \infty$.

For say a normal distribution of reservation wages it will be very close to this value for all $w > \mu_w + 2\sigma_w = \bar{w}$. $(\beta-w)$ is clearly monotonically decreasing in w .

The first order condition (4.8) is obviously highly non linear and so will not easily yield analytic solutions. However, information about the determinants of the profit maximising wage offer can be recovered via a comparative statics exercise.

For some parameter z of equation (4.8)

$$(4.9) \quad \frac{dw}{dz} = \frac{-\phi_z}{\phi_w}$$

where $\phi_w = \frac{\partial^2 E(\pi)}{\partial w^2} < 0$ by the second order condition.

The main parameters of interest in this very simple model are β (the value product), γ (the contact rate of job searchers), μ (the exogenous leaving rate) and parameters of $G(w)$.

Starting first with the parameter β which represents the marginal revenue product of labour, it can be seen directly from (4.8) that $dw/d\beta > 0$. Hence the firm's (monopsonistic) wage offer is increasing in labour productivity. Following a demand shock such that the price of output rises we would expect to observe an increase in wages and hence steady state employment. The intuition behind this result is simple. An increase in β leads to an increase in the opportunity cost of an unfilled vacancy; ceteris paribus we would expect a reduction in the proportion of time that vacancies remain unfilled. The only avenue open to firms in our simple model is via wages so that increases in β are synonymous with increased wages.

If we now consider the parameter μ which represents the extent of turnover in our model, then differentiating (4.8) with respect to μ we obtain

$$(4.10) \quad \phi_{\mu} = \frac{(\beta-w)\gamma g(w)}{[\mu+\gamma G(w)]^2} > 0$$

Hence $dw/d\mu > 0$ so that increased turnover leads to higher wage offers. This implies that firms respond to an unfavourable increase in turnover by partially offsetting the employment consequences of this change using higher wages.

The parameter γ represents the contact rate of job searchers at the firm. We might therefore expect the consequence of an increase in γ to be similar to those of a fall in μ . This is indeed the case as can be seen by differentiating (4.8) with respect to γ .

$$(4.11) \quad \phi_{\gamma} = \frac{-(\beta-w)\mu G(w)g(w)}{[\mu+\gamma G(w)]^2} = -\mu/\gamma^2 \phi_{\mu} < 0$$

Hence $dw/d\gamma < 0$.

Our results concerning the effects of changes in the parameters γ and μ are intuitive. If for example an increase in γ is considered as being synonymous with an increase in steady state unemployment (assuming a fixed number of firms) then our results suggest that there will be downward pressure on wages. Notice that care is needed, however, before drawing conclusions for employment since the two changes have opposite effects on $p(1)$. An increase in γ suggests that $p(1)$ increases, the resultant wage change has the opposite implication.

If we now consider the effects of changes in the form of the distribution of reservation wages $g(r)$ very little can be said. The effects of both mean shifts and increased risk (in the Rothschild-Stiglitz (1970) sense) are ambiguous. We can only suggest the consequences on wage offers for certain special cases.

In order to clarify discussion, let $\bar{r} = \int_0^{\infty} rg(r)dr$

and define σ as the measure of riskiness of $g(r)$.

Firstly considering shifts in \bar{r} with σ fixed. As \bar{r} increases we cannot systematically sign the consequence for $g(w)$ (of course $G(w)$ falls as \bar{r} increased). Hence the partial derivative $\phi_{\bar{r}}$ is ambiguous in sign.

In the case of a mean preserving spread of $g(r)$ we can draw conclusions if the solution to equation (4.8) lies 'close' to \bar{r} (i.e. if $w = \bar{r} + \epsilon$ or $w = \bar{r} - \epsilon$). To see this it is useful to bear in mind that a Rothschild-Stiglitz increase in risk implies that $G(w)$ increases if $w < \bar{r}$. Diagrammatically an increase in risk of a symmetric distribution has consequences for the distribution function as shown in Fig. 2.

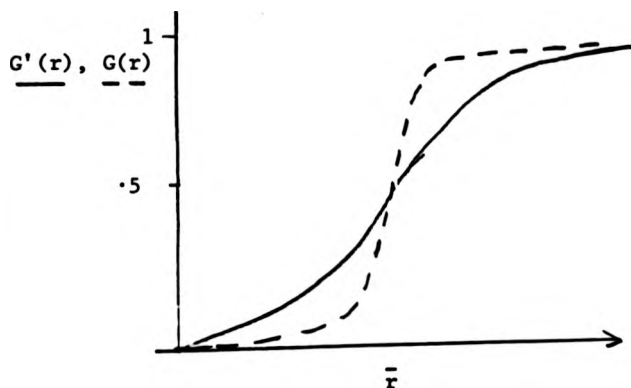


Figure 2

$G'(r)$ is here drawn more risky than $G(r)$. An increase in σ will also result in a lower value for $g(w)$ provided w is sufficiently close to \bar{r} .

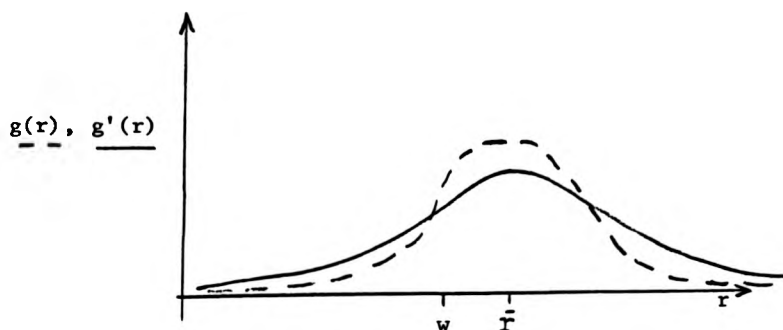


Figure 3

Hence in this particular case $\left. \phi_{\sigma} \right|_{w=\bar{r}-\epsilon} < 0$ thus $dw/d\sigma < 0$. In

this special case an increase in the riskiness of searchers' reservation wages leads to a reduction in the firm's profit maximising wage offer. Clearly this result is not general and if $w > \bar{r}$ or if w is not sufficiently close to \bar{r} an increase in risk has ambiguous consequences for w .

This section has therefore analysed the comparative statics of wage determination of the single vacancy firm. Our model is one where a firm can control its inflow of applicants by setting its wage offer. The higher the offer the greater the proportion of randomly contacting job searchers who will find it acceptable. We have seen that the profit maximising wage offer is increasing in the value product of labour and the degree of labour turnover and decreasing in the contact rate. We could not predict the consequences of changes in the mean or riskiness of the reservation

wage distribution $g(r)$. The firm we have considered is the most simple consistent with the generation of null offers. Indeed we can draw some conclusions from our analysis for the determination of the null offer probability that featured as an important determinant of individual search behaviour in chapters 2 and 3. It is worthwhile pausing here to consider the predictions of the model developed here. The null offer probability is given by $p(1)$ so that our parameter q in chapter 2 is here given by $(1-p(1))$. Only therefore in the case of the parameter β have we an unambiguous prediction concerning the effect on q . As β increases the firm wage offer and $p(1)$ increases, ceteris paribus q falls and according to our earlier analysis individual search behaviour will be affected through both wage and offer probability effects. This will presumably be reflected in a change in $g(r)$ and hence the market equilibrium effects will work through. The model presented here does not therefore suggest itself as very useful in examining the equilibrium consequences of offer rationing, since we have been unable to suggest the likely consequences either for the null offer probability or for wages of changes in individual behaviour. For this reason our own equilibrium analysis in chapter 6 will simplify the model analysed here by considering a degenerate distribution of reservation wages.

In the remainder of this chapter we shall be concerned with further analysis of our framework by considering the consequences of relaxing some of the special assumptions that have been made.

4.5. The Infinite Vacancy Firm

The model of section 4.4. considered a firm's profit maximising wage offer when there existed a single job slot to fill. In this section we consider the polar opposite case of a firm able to expand employment without limit. In terms of the two interpretations offered above we are now considering either a firm for which capital stock does not limit employment or one which faces no constraint with respect to sales of output. The central question addressed here therefore concerns the consequences for wage determination of unlimited vacancies.

With $L = \infty$ it is natural to look for a simplification on technology that allows only one (as opposed to an infinity) feature of the employment process to be considered. We have already discussed the role of constant returns (linear technology) in this respect. Rather than look at steady state probabilities we therefore consider only expected employment in steady state, and derive an expression for expected employment.

If the same wage is paid independent of employment, the employment process has again two parameters, λ (a function of w) and μ (exogenous to the firm).

A process such as this which grows without limit and has just two parameters is considered in Cox and Miller (1970), p. 356. The 'expected' state is simply given by

$$(4.12) \quad \bar{\ell} = \frac{\lambda(w)}{\mu}$$

Since each individual leaves with probability μ the larger the firm grows the greater the rate of loss, which therefore acts as a natural limit to firm size. Again the expected result that employment increases in w (and is bounded by $\frac{\gamma}{\mu}$) and decreases in μ (bounded by zero) can be seen immediately. Expected profit can now be written as

$$(4.13) \quad E(\pi) = \frac{(\beta-w)\lambda(w)}{\mu} - \bar{k}$$

and wage determination examined by considering the f.o.c.

$$(4.14) \quad \frac{(\beta-w)g(w)\gamma}{\mu} = \frac{G(w)\gamma}{\mu}$$

In contrast with the single vacancy case the firm's wage offer is independent of γ or μ . As before, an increase in β unambiguously increases w , whilst in general shifts $g(w)$, $G(w)$ are ambiguous in effect.

The most interesting result concerns the simplification with respect to γ and μ . When vacancies can be created 'costlessly' the parameters γ and μ enter into the two elements of 'marginal' profit (profit per vacancy and marginal employment) in the same way and hence become irrelevant for wage determination.

If one once again thinks of γ as reflecting the slackness of the labour market, (i.e. Unemployment), then in this special case such slackness does not affect wages. It is therefore clear that an unrationed vacancy world is inconsistent, at least at the micro level, with explanations of the Phillips relationship.

Away from such an extreme assumption it is clear that our analysis of the 'single vacancy' firm can without difficulty be generalised as the 'L vacancy' firm. Again it follows that γ and μ will be important in determining the firm's wage offer but that their effect may be ambiguous. Hence when it comes to chapter 5 we will have an idea of the sort of factors determining wages in a search setting even if we cannot sign parameter changes. There are, however, two strong assumptions made above which must now be examined, namely those that wages paid do not depend on employment and that the future is not discounted.

4.6. State Contingent Wages, The Two Vacancy Firm

So far we have ruled out by assumption the possibility of making wage offers dependent upon the employment state occupied. In order to formally examine this question in the simplest framework possible, we can extend the model of section 4.4. and consider a two vacancy firm.

Again we shall assume constant returns and the expected per period profit maximisation framework but now allow for the firm to pay a different wage when fully employed ($l = 2$) say w_2 to that period when under employed ($l = 1$), w_1 .

First we need to derive the steady state probabilities given wages w_1 , w_2 and then examine the maximising choices of a firm with respect to these.

The employment process is pictured in Fig. 4 and from previous discussion we have the following relationships between the steady state probabilities

$$(4.15) \quad \lambda_1 p(0) = \mu p(1)$$

$$\lambda_2 p(1) = \mu p(2)$$

$$p(0) + p(1) + p(2) = 1$$

Where $\lambda_1 = \lambda(w_1)$ the probability of gaining an employee given current 0 employment and an offer w_1 .

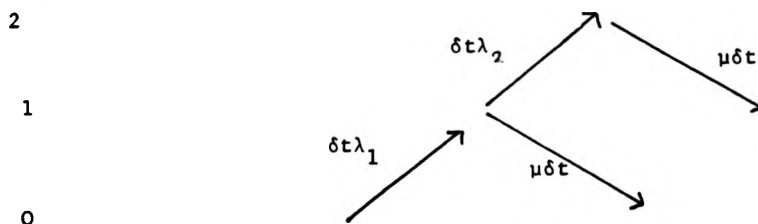


Figure 4

Solving equations (4.15) yields

$$(4.16) \quad p(0) = (1 + \lambda_1 / \mu [1 + \lambda_2 / \mu])^{-1}$$

$$p(1) = [\lambda_1 / \mu] p(0)$$

$$p(2) = [\lambda_2 / \mu] p(1)$$

Notice now that all state probabilities depend upon both w_1 and w_2 and that $p(1)$, $p(2)$ are increasing in both w_1 and w_2 .

There is now a problem when we come to consider expected steady state profit since there exists at present a possibility that a firm offering w_1 whilst in state 0 will wish to revise and offer w_2 if an individual accepts (w_2 being the profit maximising wage contingent on state 1).

We will therefore assume that the firm makes a binding commitment to pay a wage of w_2 when in state 2 and w_1 when in state 1. A worker who contacts the firm when it is in state 1 will know that with his accepting the firm's wage offer w_2 the firm will move to state 2 and that thereafter his wage payments will vary with the firm's employment state.

Hence profits in steady state can be written:

$$(4.17) \quad E(\pi) = (\beta - w_1)p(1) + (\beta - w_2)p(2) - \bar{k}$$

When in state 1 (which accounts for a proportion of $p(1)$ of each period) a firm employs a single individual who must have been recruited from state 0 and hence is paid w_1 . When in state 2 the firm has two employees to which it pays w_2 .

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The first question we can address is whether making wage payments contingent upon employment state is generally preferred by the firm to state independent wages.

Maximisation of (4.17) implies that the following first order conditions are satisfied

$$(4.18a) \quad -p(1) + p_1(1)(\beta - w_1) + 2(\beta - w_2) p_1(2) = 0$$

$$(4.18b) \quad -p(2) + 2p_2(2)(\beta - w_2) + (\beta - w_1) p_2(1) = 0$$

In equations (4.18) $p_1(1)$ denotes $\partial p(1)/\partial w_1$. For a unique wage offer w^* to satisfy equations (4.18) simultaneously we require that (conditional on $w_1 = w_2$) $p(1) = p(2)$, $p_1(1) = p_2(1)$, and $p_1(2) = p_2(2)$. Referring back to equations (4.16) we can express the partial derivatives (conditional on $w_1 = w_2$) and hence $\lambda_1 = \lambda_2$ of the steady state probabilities as

$$\begin{aligned} p_1(1) &= [\lambda_w/\mu]p(0) + [\lambda/\mu]p_1(0) \\ p_1(2) &= [2\lambda\lambda_w/\mu^2]p(0) + [\lambda/\mu]^2 p_1(0) \\ p_2(1) &= [\lambda_w/\mu]p(0) + [\lambda/\mu]p_2(0) \\ p_2(2) &= [2\lambda\lambda_w/\mu_2]p(0) + [\lambda/\mu]^2 p_2(0) \\ p_1(0) &= p_2(0) = -(\lambda w/\mu + \frac{2\lambda\lambda_w}{\mu_2}) \cdot p(0)^2 \end{aligned}$$

It is therefore clear that $p_1(1) = p_2(1)$ and $p_1(2) = p_2(2)$. However $p(1) \neq p(2)$ except in one special case where $\lambda/\mu = 1$. Since λ is endogenously determined by the firm's choice of w it will only be by pure chance that both equations (4.18) will be solved by a common w .

We can therefore conclude that in general state dependent wages will be preferred by the firm. It cannot be possible for the maximum profit obtained by choosing w_1 and w_2 unconstrained to be less than the profit obtained under the constraint $w_1 = w_2$. It is apparent from the above that only under exceptional circumstances will the outcome of the two problems (constrained and unconstrained) be equal.

The second question which we can ask is whether wages increase or decrease in the level of employment. Intuition here suggests the latter possibility. As the firm approaches full employment of its available capital we might expect it to be less concerned with increasing employment further. However, consideration of the first order conditions of our model suggest that no general result of this nature are possible.

Consider once again equations (4.18) when $w_1 = w_2 = w^*$. Two situations are possible. When $p(2) > p(1)$ (evaluated at w^*) $(4.18a) > (4.18b)$ and vice versa. We can use this fact together with the second order conditions for profit maximisation to deduce the sign of $w_2 - w_1$. The second order conditions require the derivatives of (4.18a) and (4.18b) with respect to w_1 and w_2 be negative.

Hence when $p(2) > p(1)$ we can deduce that in order for equation (4.18) to be simultaneously satisfied $w_1 > w_2$. In this case therefore the intuition that wages decrease in employment. Notice that $p(2) > p(1)$ when $\lambda_2/\mu > 1$ where of course $\lambda_2 = \gamma G(w_2)$. The inequality is therefore likely to hold ceteris paribus when γ is large relative to μ .

We cannot, however, rule out the possibility that $p(2) < p(1)$ at the firm's optimal choice of w_1, w_2 . In this case the wages increase in employment. Our conclusions are therefore ambiguous.

It is not difficult to find intuition for these results. In the case where arrivals are frequent in comparison with departures the firm reduces its wage offers as employment rises. As full employment of available capital approaches the firm becomes less anxious to fill vacancies and prefers instead to reap a large profit from already filled vacancies. Such a firm by nature of its environment spends most of the time near full employment. Conversely a firm which faces frequent departures and few arrivals chooses to make a large profit per vacancy whilst in low employment states.

The possibility of state contingent wages obviously greatly complicates analysis. We can deduce for example that both w_1 and w_2 are increasing in β . Other comparative statics results however are not possible. In chapter 5 when we return briefly to wage setting we shall therefore assume state independent wages. In practice such an assumption might well be realistic for reasons not modelled here. In making offers to 'new' employees firms may be constrained by existing wage structures within the firm. Economic theory has already elaborated many roles for wage structures and incentives to 'efficient' human capital investment (see Becker (1975) Gronau (1971)) and these considerations may dominate any desire by firms to alter wages in response to random employment fluctuations.

There is also a sense in which the time consistency issue noted above comes to the defence of a model which does not allow employment state contingent wages. The problem with conditioning a wage

offer upon states is that it gives an incentive when a different state arrives to renegotiate wages. Of course, in a more realistic setting there are many factors which may change a firm's perception of the best wage offer, perhaps far more important in this respect than the state of recruitment are product demand considerations. Recently contract theory has been applied to suggest circumstances under which wage agreements between workers and firms may arrive at wages paid independently of the state of the world (see Hart (1983)). This literature would seem to offer some support for the notion of state independent wage offers, which will form the basis of analysis in Chapter 5.

4.7. The Role of Discounting

In the analysis of this chapter so far we have ignored the role of discounting and concentrated on expected per period values. This is indeed a necessary simplification if one wishes to endogenise the decision of firms to create possibly many vacancies. We can, however, examine the role of discounting in a very simple framework in order to deduce its effect in more general settings.

In this section we revisit the simple single vacancy firm of section 4.3., consider a discrete time analogue and allow for discounting.

Letting $\delta t \rightarrow 1$ we can consider the value to a single vacancy firm of the profit stream,² when random matching and quitting are

2. This approach is somewhat unsatisfactory since in discrete time there is nothing to rule out multiple contacts to the firm. However, for illustration purposes it suffices.

a part of the environment. Immediately we can write

$$(4.20) \quad \begin{aligned} (1+\tau)V(0) &= -\bar{k} + \lambda V(1) + (1-\lambda)V(0) \\ (1+\tau)V(1) &= \beta - w - \bar{k} + \mu V(0) + (1-\mu)V(1) \end{aligned}$$

In equations (4.20) τ is the discount rate (end of period discounting is assumed) and $V(0)$, $V(1)$ the discounted expected profit of a firm commencing with an unfilled and a filled vacancy respectively.

In order to find an explicit solution to the firm's profit maximising wage offer it is first necessary to solve equations (4.20). This can most easily be done by writing (4.20) in matrix form so that

$$(4.21) \quad \begin{vmatrix} \tau + \lambda & -\lambda \\ -\mu & \tau + \mu \end{vmatrix} \begin{vmatrix} V(0) \\ V(1) \end{vmatrix} = \begin{vmatrix} -\bar{k} \\ \beta - w - \bar{k} \end{vmatrix}$$

The solution of equations (4.20) can therefore be written as $A^{-1}b$ where A^{-1} is the inverse of the matrix $\begin{vmatrix} \tau + \lambda & -\lambda \\ \mu & \tau + \mu \end{vmatrix}$ and b is the vector on the right hand side of (4.21). A^{-1} can be written as

$$(4.22) \quad A^{-1} = \frac{1}{\epsilon} \begin{vmatrix} \tau + \mu & \lambda \\ \mu & \tau + \lambda \end{vmatrix} \quad \text{where } \epsilon = \tau(\tau + \lambda + \mu).$$

We are most interested here in analysing the effects of discounting on the wage offer a firm chooses. The firm sets a wage to maximise $V(0)$

We once again have a potential problem in that the firm will desire to make a high wage offer to induce an applicant to accept and then actually pay low wages when the offer is accepted. Again we rule such behaviour out by assumption and analyse the wage choice that maximises $V(0)$ above.

To examine the effect of the discount rate upon the wage we again use the fact that

$$(4.23) \quad \text{sign } \frac{dw}{d\tau} = \text{sign } V_{w\tau}(0)$$

From equation (4.22) we know that $V(0)$ can be written

$$(4.24) \quad \varepsilon V(0) = -(\tau + u) \bar{k} + \lambda(\beta - w - \bar{k})$$

Differentiating (4.24) with respect to w and τ and rearranging we obtain

$$(4.25) \quad \varepsilon_{w\tau} V_{\tau}(0) = - (\varepsilon_{-w} V(0) + V_w(0) \varepsilon_{\tau} + \varepsilon_w V_{\tau}(0))$$

On the right hand side of (4.25) we have three terms, the middle one of which must be zero by the first order condition for profit maximisation. By inspection the first term $(\varepsilon_{-w} V(0))$ is positive whilst the third term $(\varepsilon_w V_{\tau}(0))$ is negative.

However, if we differentiate (4.24) with respect to τ we obtain an implicit expression for $V_{\tau}(0)$, namely

$$(4.26) \quad V_{\tau}(0) = - (k + \varepsilon_{-w} V(0)) / \varepsilon$$

Using (4.26) we can rewrite (4.25) as

$$(4.26) \quad \varepsilon_{w\tau}^2 V_{\tau} = V(0) [\varepsilon_{-w} \varepsilon_{\tau} - \varepsilon \varepsilon_{w\tau}] + \varepsilon_w k$$

Differentiation of ε as defined in (4.22) results in the observation that

$$(4.27) \quad (\varepsilon_{w\tau} - \varepsilon_{\tau w}) = \tau \lambda_w (2\tau - \tau) = \tau \lambda_w > 0$$

Hence since ε is positive we can deduce that $V_{w\tau}(0) > 0$ which in turn implies that $dw/d\tau > 0$. An increase in the discount rate induces the firm to make higher wage offers.

The intuition behind this result is easily seen. Incorporating discounting into our analysis allows for an impatience effect. We have assumed that the firm sets its wage offer from the vacant state. Impatience in this case implies that the firm attaches a cost to the time it takes to find a suitable employee. The higher is this cost (the larger is the discount rate) the more the firm will be prepared to pay in order to attract a suitable worker more quickly. Conversely, as the rate of time preference of the firm becomes smaller it becomes less important to fill a vacancy now rather than later. In the limit we move to the case of a zero discount which we have analysed in section 4.4.

4.8. Wage 'Bargains' and Wages

It might be objected that the discussion of this chapter has proceeded along the lines of unilateral wage offers by firms, whereas in a market characterized by imperfect information all agents have 'monopoly' power. Just as a firm once contacted is different to the individual from other firms yet located, so is that individual different to the firm from others yet to find the firm. This idea of bilateral monopoly in a search setting is prevalent in the literature (see for example Diamond (1982)) and will indeed be used in Chapter 6.

The main conclusions we have drawn from our analysis are, however, concerned with only one side of this bilateral monopoly problem. These conclusions remain valid when wage bargaining is allowed since what our analysis has really determined is one extreme of the contract curve along which bargained outcomes must lie.

The work of Nash (1960) on bargaining outcomes suggests a way of choosing points from a set of possible bargaining solutions all of which must be privately efficient.³ At one extreme of the set lies one agent's 'ideal' or most preferred outcome and this is precisely what we have derived above for the case of a bilateral worker/firm wage bargain in a search setting. There is some rather loose justification for concentrating on the firm's wage choice rather than any bargained outcome if one is prepared to believe that firms are able to extract the most surplus on account of their size vis a vis individuals. There is also the possibility that unilateral wage offers rather than wage bargains will predominate in situations of asymmetric information. In the setting we have modelled above it would seem reasonable to regard an individual's reservation wage as 'private' information something that the individual knows and the firm cannot observe.

Even without such beliefs, however, the central insight of this chapter carries through since a knowledge of the end points of the contract curve is a pre-requisite for an analysis of a wage bargain. We have established how a firm's optimal wage offer will vary in response to changes in the economic parameters that naturally characterise a search setting.

3. That is to say must exhaust the gains of trade between the agents concerned, but need not be socially desirable.

4.9. Summary and Discussion

This chapter has concentrated on the wage offer decisions of firms in a simple constrained vacancy setting. We have examined in some detail a method of modelling firms in a search environment which entails considering employment as following a simple stochastic process governed in part by a firm's decisions. To capture the notion of 'limited' vacancies we considered a single vacancy firm, but also allowed for some generalisation and in particular briefly considered an 'infinite' vacancy firm.

Emphasis has been laid on the fact that in a search setting it is no longer reasonable to think of the wages that motivate individuals search decisions analysed in chapters 2 and 3 as being competitively determined by the forces of supply and demand. Instead we have chosen to model the determination of wages when firms are able to make choices about the compensation to labour. Since we are most interested not only in 'search' models but in search models where individuals may be unsuccessful when contacting firms we built null offers into the model examined here in a very simple way by exogenously restricting the vacancies on offer at the firm.

Taking the simplest of all cases we found that it was possible to unambiguously determine the effect upon wage offers of certain crucial labour market and firm specific variables. The profit maximising wage offer was increasing in the value product of labour and the exogenous separation rate (μ) and decreasing in the contact rate (γ). No general results were possible, however, concerning shifts in the distribution of workers reservation wages that the firm faces.

These results suggested that monopsony wage offers respond in an intuitive fashion. In a search setting we have argued that the firm's wage offer regulates the rate at which vacancies are filled. The faster unemployed searches contact the firm the lower the wage required to maintain employment.

In the remainder of the chapter we attempted to add to these results and check their robustness to the simplifying assumptions made.

Moving away from the single vacancy firm towards an 'infinite' vacancy setting it was shown that the rate of flow into and away from the firm became in this very special case irrelevant to wage determination. Since unlimited vacancies implies the impossibility of null offers the infinite vacancy firm is not of particular further interest except that we shall use the model developed to highlight the equilibrium consequences of offer rationing itself.

In the case of multiple (finite) vacancy firms there exists the possibility that firms will condition wage offers on their employment condition. We demonstrated for one case that indeed there did exist an incentive to make state contingent wage offers. This considerably complicates analysis and does not correspond well with casual empirical observation. There is good reason to believe that many factors in labour markets constrain wage offers to be made independent of the individual firms employment condition.

Finally, we relaxed our no-discounting assumption and succeeded in deriving some intuitively appealing results concerning the effects of discounting. For a firm commencing from a position of unfilled vacancies an increase in the discount rate leads to an increase in

wage offers as firms become impatient to fill their vacancies. For a firm starting with a filled vacancy the effect of an increase in the discount rate is the opposite of the above. Where the future matters little such a firm will attempt to maximise the return from its already filled vacancy having little regard for future employment consequences.

In many ways this chapter has been a first attempt at a problem worthy of considerably more analysis. The early search literature was very concerned with applying search ideas to such macroeconomic issues as the unemployment/inflation trade off (see Phelps (1970)) for example). At the centre of this analysis lies the question of how wages respond to changes in the unemployment rate as this proxies the state of labour demand and supply. We would suggest that the analysis of this chapter might have a role to play in this strand of what has become known as the 'micro-foundation' literature. Our main concern however lies in the analysis of search in labour markets characterised by null offers and with this view in mind we leave the issue of wage determination in order to consider the actual generation of vacancies.

Chapter 5

Job Search and the Firms' Decisions

2: Vacancy Creation

5.1. Introduction

In chapter 4 we considered issues related to a firm's wage offers in markets characterised by search. In order to capture the notion of offer rationing or null offers we dealt with a model of the single vacancy firm. It has already been noted that a constraint on the number of job offers a firm makes may be imposed exogenously by limits on sales of a perishable output; in this chapter we wish to consider the endogenous determination of such constraints, brought about by the costly nature of vacancy creation.

The interest in the determinants of the number of vacancies in a search environment arises because of the policy relevance of this topic. Recently many writers including Layard et al (1984) have considered ways of influencing the employment decisions of firms in order to generate more vacancies. A word of caution is, however, needed here. In order to proceed with our analysis we shall have to make some strong assumptions which considerably reduce the ability of the model developed here to answer policy orientated questions. The reason why such strong assumptions are necessary is because we wish to deal explicitly with job search as a phenomenon and this, as has been already noted, entails a consideration of both stochastic and dynamic problems. In order, therefore, to consistently model the role of search parameters in a firm's vacancy creation decisions we must abstract considerably from reality. The hope when performing such an exercise is that some insights are gained which may prove useful components of more policy orientated, less pedantic models of labour market operations.

As in chapter 4, the framework for analysis adopted here involves consideration of the firm's employment following a

stochastic process. We shall assume away the wage determination problem in the first instance so that the only decision a firm has to make is with regard to the number of job slots or opportunities to create. This 'cut-off' point of course determines the pattern of employment the firm can effect and hence the firm faces a conventional cost-benefit appraisal. Vacancy creation will in general ensure greater employment but will also be costly. We wish to examine the ways in which this trade-off depends upon the search parameters, which we have previously identified as being the rate of contact (γ) and the leaving rate (μ).

The final section of this chapter is taken up by a brief consideration of the likely implications of firms' decisions on market equilibria, a topic which is the explicit concern of chapter 6.

5.2. A General Model of the L Vacancy Firm

Consider as in chapter 4 a firm which in continuous time faces a stochastic flow of applicants (probability of contact = $\gamma \delta t$) and stochastic separations (probability of loss = $\mu \delta t$). In contrast to chapter 4, however, we now assume that all individuals have of common reservation wage \bar{r} so that

$$(5.1) \quad \lambda(w) = \gamma \quad \forall w \geq \bar{r} \\ = 0 \quad \forall w < \bar{r}$$

Hence the firm's profit maximising wage offer is simply \bar{r} .

As before, we shall limit our attention to steady states of the resultant employment process and consider the firm to maximise expected per period profit. The firm is now able to decide how many job slots (L) to create and this decision in turn affects the steady state probabilities of different employment levels. In our previous notation we have

$$(5.2) \quad E(\pi) = \sum_{l=1}^L p(l) (\bar{p}Q(l) - \bar{r}l) - k(L)$$

The only new term in (5.2) is $k(L)$ which is assumed to be the vacancy (job slot) cost function. For the time being the only restriction we shall place on this is $k_L > 0$. $k(L)$ is consistent with the idea that production requires capital or 'machines' which is a perfectly complementary input. In other words imagine exactly one man to a machine, then the number of machines installed determines the maximal productive employment that a firm can offer. The dependence of each $p(l)$ on the choice of L has been suppressed, but should be borne in mind.

Maximisation of (5.2) requires that L be chosen so as to equate the marginal cost of a job slot k_L , with the marginal change in revenue product. Of course strictly speaking L is an integer variable so that discrete methods are required. However, it is usually possible to think in terms of a continuous analogue and there are no real conceptual problems here.

What therefore can be said of the choice of L ? We need to know exactly how L affects each of the $p(l)$ in the first instance, but we can reasonably expect from a consideration of the stochastic process detailed in chapter 4 that that total revenue net of wage

costs is increasing for at least some L . Given that k_L is positive, this will be the only range in which the firm would choose to produce anyway. It would also be reasonable to expect an increase in $\gamma(\mu)$ to increase (decrease) total revenue for all feasible L . However, in order to say anything at all about the determination of L we need to know how a parameter affects the marginal effect of a change in L . We therefore need to consider precisely how L affects the probability distribution of employment states and payoffs to the firm. It is easiest to determine the consequences of marginal changes in L upon total revenue when there exists a closed form expression for the latter. This necessitates an assumption of constant returns to labour which we will apply in the first instance.

5.3. Employment Probabilities and Vacancy Creation

Whereas the simple stochastic process for the single vacancy firm is one frequently analysed in the literature of stochastic processes (see Cox and Miller (1970)) an equivalent to what we have termed the 'L' vacancy firm does not appear. We shall first therefore derive the steady state employment probabilities.

Denoting by $P(l, t)$ the probability of being in state l at time t , the discussion above suggests the following relationships

$$(5.4) \quad P(l, t + \delta t) = \lambda \delta t P(l-1, t) + \mu \delta t P(l+1, t) + (1 - \lambda \delta t - \mu \delta t) P(l, t)$$

$$\text{for } 0 < l < L$$

$$(5.5) \quad P(0, t + \delta t) = \mu \delta t P(1, t) + (1 - \lambda \delta t) P(0, t)$$

for $l = 0$

$$(5.6) \quad P(L, t + \delta t) = (1 - \mu \delta t) P(L, t) + \lambda \delta t P(L-1, t)$$

for $l = L$

Equations (5.4) - (5.6) are simply statements relating to the time dependency of state probabilities. We now wish to move to steady states, so transposing terms of $P(l, t)$, $P(0, t)$ and $P(L, t)$ dividing by δt and taking the limit as $\delta t \rightarrow 0$ yields three differential equations.

$$(5.7) \quad P_t(l, t) = -(\lambda + \mu) P(l, t) + \lambda P(l-1, t) + \mu P(l+1, t)$$

$$(5.8) \quad P_t(0, t) = -\lambda P(0, t) + \mu P(1, t)$$

$$(5.9) \quad P_t(L, t) = -\mu P(L, t) + \lambda P(L-1, t)$$

In steady state by definition $P_t(l, t) = 0 \quad \forall l$ so that (5.7) - (5.9) imply by recursion that

$$(5.10) \quad \begin{cases} p(1) = p(0) \cdot \frac{\lambda}{\mu} \\ p(2) = p(0) \left(\frac{\lambda}{\mu}\right)^2 \\ p(l) = p(0) \left(\frac{\lambda}{\mu}\right)^l \end{cases}$$

Finally, we require that probabilities sum to unity so that $p(0)$ can be solved for as

$$(5.11) \quad p(0) = \left[1 + \frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu}\right)^2 + \dots + \left(\frac{\lambda}{\mu}\right)^L \right]^{-1}$$

Equations (5.10) and 5.11) serve to confirm our earlier conjectures over the role of L . As more job slots are created, the probability of the highest employment state increases from 0 to $p(0)\left(\frac{\lambda}{\mu}\right)^L$ whilst the steady state probabilities of other lower states decline.

We can distinguish two cases and represent them diagrammatically (see Fig. 1). The first is where $\lambda > \mu$ so that probabilities increase in employment, the second where $\mu > \lambda$ where the reverse holds

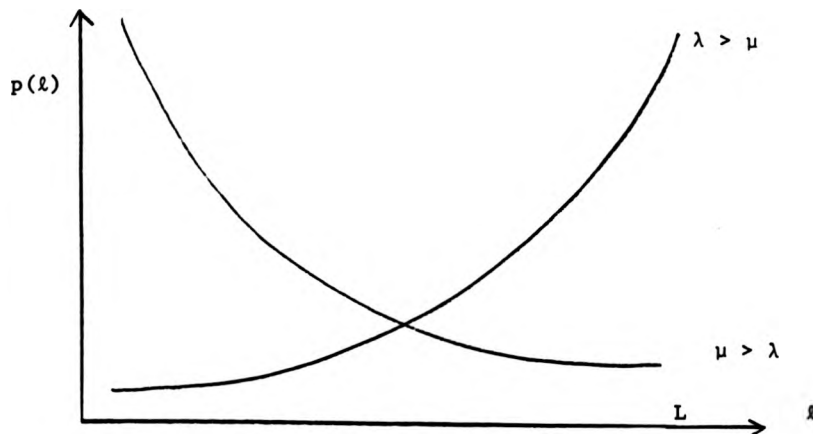


Fig. 1. Approximate Graph of Steady State Probabilities

The effect of a change in L upon the firm's total revenue is of course ambiguous. It might well be that the extra product of the L th worker obtained for a proportion $p(L)$ of each period does not compensate for the loss of some proportion of production for other employment states. We do know, however, that the firm will choose to produce in a region where revenue is increasing in L .

The parameters γ and μ also enter into the steady state probabilities in the conjectured fashion. Notice that an increase in γ reduces $p(0)$ and increases the probability of at least some high employment states, whilst μ has the opposite effect. The conjecture of equation (5.3) is therefore confirmed by our formal analysis.

Since we are concerned with an analysis of optimal decisions regarding L it is most useful to have a continuous analogue to the above discrete case. This is therefore derived as our next step.

By analogy to the discrete case (equation (5.10)) we have

$$(5.12) \quad \mu p(l + \delta l) = \lambda p(l)$$

Rearranging by subtracting $\mu p(l)$ from both sides yields

$$(5.13) \quad \mu(p(l + \delta l) - p(l)) = (\lambda - \mu)p(l)$$

$$= \frac{\delta l \mu (p(l + \delta l) - p(l))}{\delta l} = (\lambda - \mu)p(l)$$

$$= \frac{dp(l)}{dl} = \frac{(\lambda - \mu)}{\mu} p(l)$$

Solving this differential equation for $p(l)$ we obtain

$$(5.14) \quad p(l) = Ce^{bl}, \quad b = \left(\frac{\lambda}{\mu}\right) - 1$$

$$C = p(0)e^0 = p(0)$$

Where C is a constant of integration. Again we can solve for $p(0)$ by requiring probabilities to integrate to unity so that

$$(5.15) \quad \int_0^L p(0)e^{bl} dl = 1 \Rightarrow p(0) = \frac{(\lambda/\mu - 1)}{(e^{(\lambda/\mu - 1)L} - 1)}$$

Finally we shall be considering in the first instance the case of constant returns to labour, where the objective function (5.2) can be integrated directly to yield

$$(5.16) \quad E(\pi) = (\beta - w)\bar{l} - k(L)$$

We therefore require an expression for expected employment \bar{l} . Using the above derivations we obtain

$$(5.17) \quad \bar{l} = \int_0^L p(l)l dl = \frac{Le^{bL}}{(e^{bL} - 1) - (1/b)}$$

This section has served to generate the tools needed in order to conduct a comparative statics examination of the determination of L the number of job slots. Such an analysis is the concern of the next section.

5.4. The Comparative Statics of L

Differentiation of (5.16) w.r.t. L yields the following first order condition for the choice of L to satisfy

$$(5.18) \quad (\beta - w) \bar{\ell}_L - k_L = 0$$

Where $\bar{\ell}_L$ can be written

$$(5.19) \quad \bar{\ell}_L = p_L(0) \int_0^L \ell e^{b\ell} d\ell + L p_L(0) e^{bL}; \quad p_L(0) = \frac{-b^2 e^{bL}}{(e^{bL} - 1)^2} < 0$$

Using the fact that $p(0)e^{bL} = -p_L(0) \int_0^L e^{b\ell} d\ell$ (since the probabilities must sum to unity before and after a change in L) (5.19) can be rewritten as

$$(5.19a) \quad \bar{\ell}_L = p_L(0) \int_0^L (\ell - L) e^{b\ell} d\ell > 0$$

So that expected employment is increasing in the number of job slots L. This accords with intuition; the firm can influence expected employment by creating vacancies (at cost $k(L)$). The first order condition (5.18) can easily be interpreted as stating that the firm should expand the number of job slots until the marginal cost (k_L) is equal to the marginal gain $((\beta - w) \bar{\ell}_L)$.

The second order conditions for profit maximisation imply that

$$(5.20) \quad (\beta - w) \bar{\ell}_{LL} < k_{LL}$$

Differentiation of (5.19a) reveals that \bar{l}_{LL} is ambiguous in sign, hence in order to be sure that the maximisation problem is well defined we require that $k(L)$ be 'sufficiently' convex so that (5.20) holds. It might be argued that a natural specification for $k(L)$ is linear so that $k_{LL} = 0$ but this criticism need not detain us. The problem really arises because we have assumed linear production and would be alleviated if diminishing returns were assumed thus tending to make the profit function concave.

As in chapter 4 we can examine the comparative statics of the choice of L by noting that sign dL/dz for some parameter z is given by sign $|\partial f(L)/\partial z|$ where $f(L)$ is the first order condition (5.18).

It is clear that since $\bar{l}_L > 0$ (from (5.19a)) then $dL/d\beta > 0$. An increase in the value product of labour leads to the firm creating more job slots.

The other comparative statics results of interest are as in the case of chapter 4, λ and μ (notice that since $G(\bar{r}) = 1$ and $w = \bar{r}$, $\gamma = \lambda$). Results concerning λ and μ are somewhat more difficult to obtain. Differentiating (5.19a) with respect to 'b' (b is increasing in λ and decreasing in μ) we obtain.

$$(5.21) \quad \bar{l}_{Lb} = p_{Lb}(0) \int_0^L (l-L)e^{bL} dl + p_L(0) \int_0^L (l-L)L e^{bL} dl$$

From equation (5.19) we know that $p_L(0) < 0$ it is therefore necessary to sign $p_{Lb}(0)$ in order to sign the comparative statics results. Differentiating (5.19) we obtain

$$\begin{aligned}
 (5.21a) \quad p_{Lb} &= (e^{bL}-1)(2be^{bL} + \frac{Lb^2 e^{bL}}{(e^{bL}-1)^2} - Lb^2(e^{bL})^2 \\
 &= -Lb^2 e^{bL} / (e^{bL}-1)^2 < 0
 \end{aligned}$$

Since both expressions under the integrals in (5.21) are negative (since $l \leq L$) it follows that $\bar{l}_{Lb} > 0$ so that $dL / d\lambda > 0$ and $dL / d\mu < 0$.

This implies that an improvement in the conditions faced by the firm results in an increase in the number of job slots created. An improvement may take the form of either increased inflows ($\lambda+$) or decreased outflows ($\mu+$). Either of these has the effect of increasing the marginal value of an extra job slot.

The effect of an increase in the common reservation wage \bar{r} is simply the opposite of an increase in productivity β . This can be seen directly from equation (5.18). Hence an increase in wages results in a reduction in vacancies within the firm.

The decision to create vacancies we have considered here is the analogue of the labour demand decision of a firm operating in a deterministic environment. We have shown that a firm will expand the number of vacancies in response to an increase in the productivity of labour or a fall in the given wage rate. Similarly as the flow of applicants becomes greater or the outflow of employees smaller the firm will reduce the number of vacancies it creates. The vacancy creation decision we have considered is a well-defined problem even under the assumption of constant returns to labour because the stochastic process whereby vacancies are related to expected profits induces non linearity into the problem. Nevertheless the constant

returns assumption is a restrictive one since it implies the irrelevance of firm size. We therefore next consider relaxing the constant returns assumption in order to see what can be said about vacancy creation under more general production conditions.

Rewriting equation (5.2) in continuous l and differentiating we obtain the following first order condition for profit maximisation

$$(5.22) \quad \int_0^L p_L(l) (\bar{p}Q(l) - \bar{r}l) + \bar{p}Q(L) - \bar{r}L = k_L$$

Where $p_L(l) = \frac{\partial p(l)}{\partial L}$ is the effect of an increase in vacancies on the steady state probability of the l 'th employment state. Inspection of equation (5.14) reveals that $p_L(l) < 0$ for all values of l . Increasing the cut off point of employment has the effect of decreasing the probability of all lower employment states whilst adding another possible employment state. Equation (5.22) therefore has a fairly simple interpretation. The marginal cost of a vacancy comprises two parts, the first and third terms of equation (5.22). The first term accounts for the lost revenue from lower employment states that results from an increase in the number of vacancies, the third term accounts for the direct money cost of an additional vacancy. The marginal benefit is given by the second term of equation (5.22) and comprises the expected revenue derived from the L 'th employment state.

Despite the fact that we now have to consider the effect of the vacancy decision on all employment states separately rather than simply on expected employment, equation (5.22) can be simplified. Since probabilities must sum to unity it follows that

$$(5.23) \quad p(L) = - \int_0^L p_L(\ell) d\ell$$

Therefore (5.22) can be written more compactly as

$$(5.24) \quad \int_0^L p_L(\ell) (s(\ell) - s(L)) d\ell = k_L$$

where $s(\ell) = \bar{p}Q(\ell) - \bar{r}\ell$

The effect of an increase in \bar{r} can therefore be evaluated directly. Differentiating (5.24) with respect to \bar{r} we obtain

$$(5.25) \quad \int_0^L p_L(\ell) (L-\ell) d\ell$$

Now since $p_L(\ell) < 0$ and $(L-\ell) > 0$ for all values of L it is clear that (5.25) is negative, hence $dL/d\bar{r} < 0$. Our result regarding the effect of wages upon vacancy creation is therefore robust to different production technologies. Higher wages cause firms to restrict the number of vacancies.

If $Q(\ell)$ is given by a simple Cobb-Douglas production function $Q(\ell) = \ell^a$ ($a < 1$) we can use equation (5.24) to evaluate the effect of an increase in the productivity parameter a . Differentiation of (5.24) with respect to a yields

$$(5.26) \quad \int_0^L p_L(\ell) (a\bar{p}\ell^{a-1} - \bar{a}pL^{a-1}) d\ell$$

Since (5.26) is positive it implies $dL/da > 0$ so that a result analogous to our result concerning the parameter β above is also

robust in a non constant returns world.

When we come to consider the contact rate γ and loss rate μ however, we cannot unambiguously sign the effect of these parameters. Differentiating (5.24) with respect to $b = \lambda/\mu$ yields the following

$$(5.27) \quad \frac{\partial f(\ell)}{\partial b} = \int_0^L p_{Lb}(\ell) (s(\ell) - s(L)) d\ell$$

Neither term under the integral can be unambiguously signed for all values of ℓ and hence the whole of expression (5.27) is ambiguous in sign.

In order to give some kind of feel for the effects of γ and μ on the choice of L in a non constant returns world we engaged in some numerical analysis. Using a Cobb-Douglas production function as above and allowing for variation in parameter values we were able to confirm that the choice of L varies as predicted by the constant returns model. L is increasing (decreasing) in $\mu(\gamma)$ for most conceivable parameter values at least as far as the Cobb-Douglas technology is concerned. In table 1 below a sample of our numerical results are given

	L^*	μ	γ
$a = 1/2$	24	.5	.7
$a = 1/2$	29	.7	.5
$a = 3/4$	31	.5	.7
$a = 3/4$	39	.7	.5

Table 1

Sample Numerical Results¹ for Cobb-Douglas Case

These results suggest that there exists the possibility of a positive feedback effect of workers' search decisions upon vacancy creation. Taking for example the case of discouraged workers dealt with in chapters 2 and 3, it can be seen what the demand side effects of a reduction in the number of discouraged might be. The first round effect would be on our parameter γ and this in turn would lead to greater vacancy creation (L^*). By a consideration of firms' vacancy decisions we are therefore beginning an examination of the kind of interdependencies that might exist in markets characterised by imperfect information. It is important to notice that these interdependencies are not reflected in prices (i.e. the wage rate) but rather in quantities. Therefore there exists the possibility of equilibria with bootstraps properties where intervention may result in entirely new employment, vacancy and search configurations. This is a theme we shall deal with in more detail in chapter 6 (where, in fact, the entry of firms is seen as the force generating more vacancies). In the remainder of this chapter we shall consider integrating both wage and vacancy decisions and thereby integrating the analysis of this chapter and that of chapter 4.

1. See Appendix A for details of Numerical Methods.

5.5. The integration of wage and vacancy decisions

The constant returns technology example used above provides the basis for a model which allows both a wage decision and a vacancy creation decision to influence the employment outcomes of the firm. In this section we relax assumption (5.1) and return to a consideration of job searchers with a distribution of reservation wages as analysed in chapter 4. Our previous derivations of the probability distribution of employment states of course remains perfectly valid. It must now be remembered that λ is a function (through $G(w)$) of w . Since wages were shown to react ambiguously to parameters even in the most simple (single vacancy) case it should not be thought that clear results will be possible here. Rather, we are more concerned in examining whether our results concerning vacancy creation are robust to endogenising wage decisions.

We continue with the assumption that the objective of the firm is expected per period profit maximisation (the consequences of which should now be clear) and consider the problem of jointly choosing w and L .

The first order conditions of this problem can be written

$$(5.22) \quad \begin{cases} \phi_w = (\beta - w)\bar{\ell}_w - \bar{\ell} = 0 \\ \phi_L = (\beta - w)\bar{\ell}_L - k_L = 0 \end{cases}$$

The second order necessary conditions for this problem require that the Hessian of ϕ be negative definite so that

$$(5.23) \quad \phi_{ww} < 0, \phi_{LL} < 0 \quad \begin{vmatrix} \phi_{ww} & \phi_{wL} \\ \phi_{Lw} & \phi_{LL} \end{vmatrix} > 0$$

The second order partials . are

$$(5.24) \quad \phi_{ww} = (\beta-w)\bar{\ell}_{ww} - 2\bar{\ell}_w$$

$$(5.25) \quad \phi_{LL} = (\beta-w)\bar{\ell}_{LL} - k_{LL}$$

$$(5.26) \quad \phi_{Lw} = \phi_{wL} = (\beta-w)\bar{\ell}_{wL} - \bar{\ell}_L$$

We have previously defined $\bar{\ell}_L$ and $\bar{\ell}_{LL}$ notice that

$$(5.27) \quad \bar{\ell}_w = b_w \left(\frac{b^{-2} - L^2 e^{bL}}{(e^{bL} - 1)^2} \right) > 0, \quad b_w = \left(\frac{\lambda_w}{\mu} \right)$$

$$(5.28) \quad \bar{\ell}_{ww} = \frac{b_{ww}\bar{\ell}_w}{b_w} + b_w^2 \left(\frac{L^3(e^{bL} + 1)}{(e^{bL} - 1)^3 - 2b^{-3}} \right)$$

$$(5.29) \quad \bar{\ell}_{wL} = b_w \left(\frac{Le^{bL}(e^{bL}(Lb-2) + Lb+2)}{(e^{bL} - 1)^3} \right)$$

Again considering z as some parameter of ϕ a change in z will simultaneously affect the optimal choice of w and L . The signs of these changes are given by

$$(5.30) \quad \begin{aligned} \text{sign } \frac{dw}{dz} &= \text{sign } \begin{vmatrix} -\phi_{wz} & \phi_{wL} \\ -\phi_{Lz} & \phi_{LL} \end{vmatrix} \\ \text{sign } \frac{dL}{dz} &= \text{sign } \begin{vmatrix} -\phi_{Lz} & \phi_{LL} \\ -\phi_{wz} & \phi_{ww} \end{vmatrix} \end{aligned}$$

All these components can readily be signed except for ϕ_{LW} which is ambiguous. In tables 2 and 3 below we therefore report results for both contingencies ($\phi_{WL} \gtrless 0$)

z	β	γ	μ
$\frac{dw}{dz}$	+?-	+?-	-?+
$\frac{dL}{dz}$	+	?+	?-

Table 2 $\phi_{WL} < 0$

z	β	γ	μ
$\frac{dw}{dz}$	+	-?+	+?-
$\frac{dL}{dz}$	+	?+	?-

Table 3 $\phi_{WL} > 0$

A ? indicates ambiguous results, in these cases the signs given in the table indicate the results of numerical analysis conducted on the model, some examples of which are included below. Where only one sign is given this indicates that for all our numerical experiments the sign was constant.

Immediately it can be seen that nothing can be said of wage determination in the general L vacancy model. Even the result that wages increase in productivity is questioned when $\phi_{WL} < 0$ (there

is no economic interpretation of this function and therefore no a priori knowledge of its sign).

The only comparative statics results for L that are unambiguous concern the effect of an increase in β . As before an increase in the productivity of labour leads to an increase in the number of job slots.

Given the difficulty in signing comparative statics results for the case of only wage determination in chapter 4, the ambiguity of many results should come as no surprise. Given this problem it seems reasonable to use numerical experiments to attempt to confirm suggested theoretical results.

As can be seen in tables 2 and 3 the numerical experiments were most successful in the case of our L variable. It was relatively easy to specify parameters for the model that results in ambiguous results for the optimal wage, whilst, results on the optimal choice of job slots remain robust.

These results therefore serve to confirm our earlier analysis where wages were assumed (through a degenerate distribution of reservation wages) to be exogenously fixed. The result that the number of job slots is an increasing function of the contact rate seems fairly robust.

	w^*	L^*	\bar{l}	$E(\pi)$	ϕ_{wL}	a
$\beta = 60$	24	7	2.83	56.8	+	1
$\beta = 80$	38.83	18	12.18	193.5	+	1
$\beta = 100$	40.81	29	23.36	550.9	-	1
$\beta = 80$	30.92	6	3.14	76.5	+	2
$\beta = 80$	22.51	4	1.65	53.4	+	3

Table 4: Sample Numerical Results

[Notes on Table 4: For the purposes of producing some illustrative results an exponential form for $G(r)$ was chosen, other numerical experiments used a Normal distribution. In table 4 the values used are $\mu = 0.3$, $\gamma = 0.5$. Finally the parameter a is a parameter of a quadratic vacancy cost function $k(L) = aL^2$ (see Appendix for further details).]

5.6. Null Offers and Vacancy Decisions

A primary concern of chapters 2 and 3 was an examination of the consequences of offer rationing on an individual's search behaviour. Rationing of offers was modelled by allowing for a probability $(1-q)$ that search would be unsuccessful. The model discussed in this chapter allows for a firm's decisions regarding vacancy creation to determine the probability $(1-q)$. The probability that a firm randomly contacted will not possess a vacancy is in the notation of this chapter $p(L)$, the probability that a firm is in its highest employment state. In the case where the wage offer is set at \bar{r} it is the firm's vacancy decision 'L' that entirely determines $p(L)$. What then can be said of the determinants of the null offer probability?

From equation (5.10) we can write

$$\begin{aligned}
 (5.31) \quad p(L) &= \frac{1}{\left[1 + \frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu}\right)^L\right]} \dots \left(\frac{\lambda}{\mu}\right)^L \cdot \left(\frac{\lambda}{\mu}\right)^L \\
 &= \frac{\left(\frac{\lambda}{\mu}\right)^L \left(\left(\frac{\lambda}{\mu}\right)^{L+1} - 1\right)}{\left(\frac{\lambda}{\mu} - 1\right)}
 \end{aligned}$$

We know already that $p(L)$ is increasing in λ for any L and that $p(L)$ is increasing in L if $\lambda > \mu$. It therefore follows that

$$\begin{aligned}
 (5.32) \quad \frac{dp(L)}{d\lambda} &> 0 \text{ if } \lambda > \mu \\
 &\approx 0 \text{ if } \mu > \lambda
 \end{aligned}$$

In other words if a firm already has a tendency to grow more quickly than it 'dies' ($\lambda > \mu$) then a favourable shift (increase in λ say) increases the number of vacancies and increases the probability of that firm being contacted in a state of no vacancies. If $\mu > \lambda$ the effect of an increase in λ on $p(L)$ is ambiguous. As a first round effect $p(L)$ increases but then L increases and may offset this.

In this latter case we have an interesting possibility, which again hints at the possible bootstraps nature of search equilibria. As more individuals search, so both the number of vacancies and the probability of receiving an offer increase. It is this latter variable as we have seen in chapters 2 and 3 that is important in determining the participation of individuals. We therefore have the possibility of a positive feedback loop - more search activity leads to more offers, leads to more search activity.

Here we have constructed an heuristic argument. In the following chapter we shall be concerned with making precise the above suggestions.

5.7. Summary and Discussion

This chapter has been concerned with generalising the single vacancy firm model and endogenising vacancy decisions. Within the framework of stochastic employment evolution the probabilities (in steady state) of various employment levels and the relationship between these probabilities and the firm's vacancy creation decision have been derived. Vacancy costs were modelled in a simple way by considering capital and labour as complementary inputs. In this case a choice of capital stock by the firm is equivalent to a decision

concerning the maximum labour force that a firm will employ, i.e. a vacancy creation decision.

It has been demonstrated that for a constant returns to labour technology that vacancies increase with a favourable shift in conditions where a favourable shift can take the form of either an increased flow of job applicants or a reduction in the loss rate. Numerical results suggest that constant returns is not a crucial assumption for this result, neither does endogenising the firm's wage decision (as discussed in chapter 4) cause any problems, although in this case only numerical results are possible.

The greatest interest in these results stems from the suggested nature of feedback from individual search decisions back onto firm's vacancy creation decisions. It was shown that increased search activity was consistent with more vacancy creation so that supply could create its own demand. More importantly it was also shown increased vacancy creation was consistent with a reduction in the offer constraint parameter ($p(L)$) which individual job searchers face. The analysis of this chapter has therefore offered insight into the possible bootstraps nature of search equilibria.

It is worth considering briefly the implications of this and the previous chapter taken together. The usual notion of a competitive market with a wage adjusting to eliminate excess demand or supply has been completely ruled out by explicitly allowing for search and imperfect information. In a world of imperfect information the only internally consistent view of wage determination is as a bilateral bargaining game in which agents bargain over a trading surplus. This is quite unlike the usual Walrasian setting (see

Arrow and Hahn (1970)) where all excess profits are bid away. There is no reason to believe that privately bargained wages in such a world will generate the socially correct signal to agents contemplating entry or participation. This point is made explicit by Diamond (1982)). The work of both Diamond (1982) and Pissarides (1983) postulates an environment in which both firms and individuals are numerous. Where firms are few and individuals numerous the balance of power in a bargaining game will tend towards firms. Chapter 4 provides a more complete analysis than available elsewhere of the determinants of wage offers in such circumstances.

Even ignoring an active role for wages in offering incentives to entry, the work of this chapter has served to illustrate how multiple offer constrained equilibria might arise. Where one agent's decisions determine another's actions (as in the case with firms creating vacancies and individuals searching) then positive feedback from one to the other opens up just such a possibility. In this chapter the possibility of this effect has been demonstrated.

It now remains to draw together our earlier analysis and that of the last two chapters in order to discuss in an equilibrium framework the job search, offer rationing and unemployment.

Chapter 6

Market Equilibrium: An analysis of the effects
of offer rationing in equilibrium

6.1. Introduction

In previous chapters we have been concerned with analysing the consequences of quantity constraints or null offers for individual decision making and in specifying models of a firm's behaviour consistent with null offers and job search in the labour market. In this chapter we turn attention to the analysis of market equilibrium in a search context. In particular we shall be concerned with the existence of offer constraints in the labour market as an equilibrium phenomena.

It is only within an equilibrium framework that answers to many interesting questions can be attempted. Much of the early search literature, for example, explicitly noted the dangers of drawing 'comparative statics' conclusions from a partial equilibrium framework (see Rothschild (1973), Lippman and McCall (1976a)). Whilst the simple search model of chapter 2 indicated that an increase in search costs would reduce reservation wages and hence in the first instance individuals' durations of unemployment, whether this results in lower or higher unemployment depends at least in part on firms' responses to increased acceptance of offers. It is quite possible within an equilibrium setting to imagine a 'perverse' result, where an increase in search costs increased unemployment.

Besides being useful in generating predictions regarding the changes in equilibrium that result from exogenous shifts in parameters an equilibrium model can also suggest something about the desirability or otherwise of market outcomes. What ought to be as opposed to what is, is clearly a question that entails by its very nature value judgements. Most would, however, agree that if some outcome entails the possibility

of improving an agent's utility at no cost to others it is in some sense undesirable. Whether a particular market outcome is efficient in the above 'Paretian' sense is another question that may be asked of a market equilibrium model.

The greater depth of analysis possible within a market equilibrium framework is obtained at the cost of analytic complexity. This is particularly true where search is a feature of the market. The previous chapters should make clear the fact that a consideration of search activity involves modelling of decision making under uncertainty over time. In attempting to model an equilibrium in such a setting it is natural to pursue the analysis of states where, excepting random events, the world is essentially static, i.e. 'steady states'. Modelling search markets out of steady state is not impossible (see Diamond (1982)) but is very difficult and will not be attempted here.

Instead we shall be concerned with providing a consistent steady state, equilibrium model of a labour market characterised by search, unemployment and null offers. In this chapter the simplest possible case is examined which corresponds to the 'vacancy' search model as outlined in chapters 2 and 3. In other words, we will model a market within which there exists a single wage but uncertainty on the part of individuals concerning the location of firms and turnover of employment. This setting naturally gives rise to a fairly straightforward specification of the individual's search problem. In order to capture the idea of offer rationing within this setting the simple 'single' vacancy model of the firm as contained in chapter 4 is used for the demand side of the market. This rationed vacancy model is contrasted with the 'infinite' vacancy firm also discussed in chapter 4. Steady state equilibrium in both settings is characterised by an equality

between (expected) inflows to employment and (expected) outflows from employment. In the limited vacancy model search by individuals may frequently be unsuccessful and this fact will feed back onto participation decisions. With unlimited vacancies there will be no such feedback.

The idea that search equilibria will generate inefficient (too much or too little) unemployment has been explored by Diamond (1982) and Pissarides (1984). Inefficiency in search equilibrium arises out of a failure of wages to reflect the social value of agents in matching. This inefficient entry incentive problem is independent of whether offers are in any way rationed or not, we shall consider later how the inefficiency arises within the explicit model considered in this chapter.

The main purpose of this chapter is to demonstrate the importance of offer rationing in determining the nature of search equilibrium. We therefore consider a much simpler search or matching technology than that of Diamond (1982). Using this simpler notion of a matching technology we are able to compare for the first time an equilibrium model of search where null offers are a feature with a model where all search is successful.

It will be shown that the effect of rationing on search equilibrium is important and interesting. In a world of unrationed offers unemployment is in a simple model determined entirely exogenously by the parameters of the matching process. In contrast to this, offer rationing implies a multiplicity of equilibria some with low unemployment, some with high unemployment. This seems to capture an essential feature of rationed search, the idea that 'Bootstraps'

equilibria are possible, where in a model with unlimited vacancies unemployment is at a unique natural rate.

Before actually proceeding with analysis of the equilibrium search model a brief summary of the literature and of the relationship between it and the current work is in order.

The difficulty in modelling equilibrium in a search setting has led to a paucity of the literature in this area. The major contributions to date have been those of Lucas and Prescott (1974), Eaton and Watts (1977), Diamond (1982) and Pissarides (1979), (1984).

Lucas and Prescott (1974) employ the notion of a market clearing wage in order to solve for equilibrium in a set of spatially distinct markets. Unfortunately the idea of a decentralised market clearing wage seems highly inappropriate in a search context as the previous two chapters have shown.

Eaton and Watts (1977) drop the assumption of market clearing and consider firm determined wage decisions in an equilibrium model of search that defies analytic solution. Whilst this gives rise to a model which is consistent with the central idea of search the lack of analytic solution limits its usefulness.

The framework used by Diamond (1982) and Pissarides (1984) is very similar to that pursued here. Pissarides considers a single vacancy firm in discrete time and models search equilibrium in steady state. Numerous efficiency issues are considered. Diamond (1982) uses a continuous time formulation that most closely resembles the approach chosen here. There are of course detailed differences.

Whereas Diamond (1982) and Pissarides (1984) consider the incentive to entry that a search equilibrium generates, neither explicitly considers the role of offer rationing or null offers in equilibrium. That is the departure of the present work where we are able to draw on the work of chapter 4 and formulate for comparative purposes a model where vacancies are unlimited.

Towards the end of this chapter we will discuss extensions of our framework which allow one to generate a search equilibrium with wage dispersion and hence a model where null offers affect the 'how much' search decision as well as the participation decision. Our conjecture is that 'Bootstraps' type equilibria will also result in more complex models for the same reasons they arise in the simpler framework considered here. Where offers are generated in limited numbers and where this constraint affects search decisions, it is likely that equilibria are non unique and that some equilibria entail (unnecessarily) high unemployment.

6.2. Overview of the Equilibrium Model

We start by considering the basic equilibrium model of search to be detailed in this chapter. The labour market is assumed to consist of n firms and N individuals.

Each firm produces a homogenous output using labour as an input and subject to a fixed 'capital' cost. Output is sold on a perfectly competitive auction market at some price p which is considered fixed. There are two alternative simplifying assumptions regarding the generation of vacancies.

In the null offer model each firm employs at most a single individual who in conjunction with capital (the rental on which will be denoted by k) produces an output Q .

In the unlimited vacancies model each firm corresponds to the simple infinite vacancy firm of chapter 4. All randomly contacting individuals are employed by such firm and each in conjunction with capital produce Q output (so that there are constant returns to labour).

Both types of firm face job turnover as described in Chapter 4 so that employees leave the firm over time. The probability that any individual separates from the firm in a time interval δt is denoted by $\mu \delta t$.

Individuals who have separated from firms search for alternative employment. With identical individuals the only equilibrium that can be sustained is one with a unique wage offer made by all firms so that all firms pay a wage w . Individuals are assumed to know of the location of firms and to contact a firm at a deterministic rate γ_i (so that $\gamma_i \delta t$ firms are contacted in time interval δt). Where null offers are a feature of the market individuals do not know which firms currently have a vacancy therefore individuals always select a firm to contact randomly (i.e. by drawing from a hat).

The process of contacting firms (search) is assumed to involve a flow search cost c (search for an interval δt costs $c \delta t$). Where offers are unlimited the decision to participate or not simply depends upon whether the wage once obtained compensates for the period of time required to contact the firm. Where null offers are sometimes the outcome of search it also matters to the individual that some

contacts may be unsuccessful. We shall see that it is in this aspect of search decisions that positive feedback effects suggest multiple equilibria as a possibility.

Stochastic equilibrium in a market with firms and individuals as described above occurs where expected separations equal expected matching, there is no incentive to entry or exit and where the wage satisfies either the firm's monopsony offer or some Nash bargaining outcome. The possibility of a bargain over wages will be examined in detail later. It is important to bear in mind that the wage will be endogenous to search equilibrium. Associated with steady state equilibrium will be steady state unemployment.

We start our detailed exposition of the above model by reconsidering the simple vacancy search model (with turnover) in continuous time.

6.3. Unemployed Vacancy Search

As noted above, we wish to reformulate the basic vacancy search model of Chapter 2 in continuous time. For simplicity discounting will be ignored throughout this chapter.

It is easiest to start by considering the proportion of time that an individual spends unemployed when the rate of contact at firms is γ_i . Considering expected per period values is natural when discounting is ignored and an infinite horizon is assumed.

The infinite lifetime of an individual is spent alternating between just two states, employment and unemployment. We again have a simple stochastic process to solve for (see Figure 1).

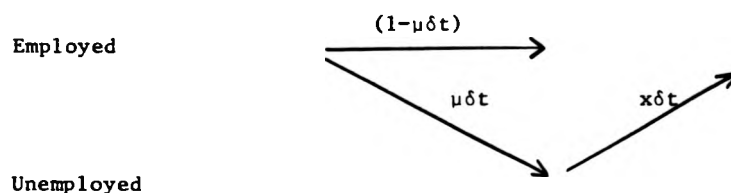


Figure 1.

The probability $x\delta t$ depends on whether offers are rationed or not. In the first case $x\delta t$ is equal to $q\gamma_1\delta t$ where q is the probability of a vacancy $((1-q) = \text{probability of a null})$ in the latter case $x\delta t$ is simply $\gamma_1\delta t$.

The steady state probabilities of this process can be solved for using the same techniques as in chapter 4 to yield

$$(6.1) \quad p(U) = \frac{\mu}{(\mu+x)}$$

$$p(E) = \frac{x}{(\mu+x)}$$

$p(U)$ and $p(E)$ are respectively the proportion of time spent in the unemployed and employed states.

When in the first of these states the individual is assumed to receive $(b-c)$ (unemployment utility minus search cost) whilst in the second a wage of w . Expected per period income given participation is thus simply:

$$(6.2) \quad V_s = (b-c)p(U) + wp(E)$$

The participation wage w^* can be calculated by equating V_s to the expected per period value of non participation which by definition is b . Hence the participation wage w^* satisfies

$$(6.3) \quad b - (b-c)p(U) - w^*p(E) = 0$$

Provided the market wage is greater than w^* all individuals will prefer participation to pure leisure.

It is important to notice that in the case of unlimited offers w^* is entirely determined by exogenous parameters whilst in the case of null offers w^* in part depends upon $(1-q)$ the null offer probability that will be endogenous to our model.

One interesting result should be noted, if $c = 0$ then $w^* = b$. This follows directly from equation (6.3) using the fact that probabilities sum to one, we shall have cause to use this special case in which (even with rationed offers) the participation wage is exogenous. On reflection the exogeneity of the participation wage in the case of zero search costs is obvious - in the absence of such costs and discounting - the probability of state occupancy is irrelevant. In such circumstances any wage greater than the value of leisure will induce participation.

When money search costs are allowed individuals must receive a wage sufficient to compensate them for unsuccessful but costly search.

The model of individual decisions outlined here is the simplest possible, allowing only one endogenous parameter ($(1-q)$ the null offer probability) to determine behaviour (participation). γ_i , u , b and c are all exogenous parameters.

In the next section we consider in detail two alternative models of firms upon which individual search decisions will feed back and hence generate a market equilibrium.

6.4. Vacancies, Profits and Wages

In this section we simply require a simplification of the 'single' and 'infinite' vacancy firms discussed in detail in chapter 4, consistent with the single wage equilibrium model.

Since all searching individuals are assumed identical the continuous distribution of reservation wages used in chapter 4 is degenerate. Hence we can simplify $G(w)$

$$(6.4) \quad G(w) = 1 \quad \forall \quad w \geq w^* \\ = 0 \quad \forall \quad w < w^*$$

The monopsonistic wage offer of all firms in such circumstances is obviously simply w^* .

Hence we can write steady state per period profit functions for the single and infinite vacancy firms as

$$(6.5) \quad \pi_s = (\beta - w^*)p(F) - k \\ \pi_i = (\beta - w^*)e - k$$

Where $p(F)$ is the steady state probability of a 'filled' vacancy and e is steady state employment.

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Where $p(F)$ is the steady state probability of a 'filled' vacancy and e is steady state employment.

To maintain consistency with our previous notation we denote by $\gamma \delta t$ the probability that a firm is contacted in the interval δt by an unemployed job searcher (the relationship between γ_i and γ' will be examined in the next section). From the results of chapter 4 we can immediately write

$$(6.6) \quad p(F) = \frac{\gamma}{(\gamma + \mu)}$$

$$e = \frac{\gamma}{\mu}$$

In the case of single vacancy firms $p(F)$ is the probability that a randomly contacting individual will find the firm with no vacancy. We therefore have the following important identity:

$$(6.7) \quad p(F) \equiv (1-q) \quad \text{the 'Null Offer' probability}$$

Total employment under the two alternative firm types can also be written directly as can steady state unemployment

$$(6.8) \quad \begin{aligned} E_s &= np(F) = n(1-q) \\ E_i &= ne \end{aligned}$$

$$(6.9) \quad \begin{aligned} U_s &= N - E_s \\ U_i &= N - E_i \end{aligned}$$

In order to complete our detailed exposition of the equilibrium search model we next consider the process whereby unemployed individuals and firms meet. In doing this we establish the relationship between

the probability that a firm is contacted (γ) and the rate at which unemployed individuals contact firms (γ_i).

6.5. The Matching Process

As has been noted by both Diamond (1982) and Pissarides (1984), a well specified model of matching processes in markets where one or both sets of agents search randomly has not yet been developed. In the absence of such a matching model it is usual to assume a contact function and specify in general terms some of its properties. If there are U searching individuals and a possible n firms for them to contact we might expect the number of matchings that occur during a unit time interval to depend positively on both U and n . Denoting by $M\delta t$ the number of matchings occurring in time interval δt it is usual to write

$$(6.10) \quad M = m(U, n), \quad m_U \geq 0 \quad m_n \geq 0$$

It is far from clear, however, whether $m(U, n)$ should be concave or convex in either of U or n , or whether $m(U, n)$ possesses homogeneity of any particular degree. In this latter respect it is perhaps most reasonable to assume that a doubling of U and n simply doubles the number of matchings that will occur. $m(U, n)$ may be viewed as a kind of 'contact' production function which, if the above restriction applies, exhibits constant returns.

The model of this chapter is based upon a far simpler specification of the matching technology. We assume that search by individuals is deterministic except that the firm to be contacted is selected at

random. This specification appears a reasonable representation of labour market search processes where individuals often know the whereabouts of firms but do not know which particular firms have vacancies.

We therefore explain the number of matchings occurring by first defining the number of contacts between workers and firms. In line with the argument above, the total number of contacts will be considered a simple linear function of unemployment, hence we write

$$(6.10a) \quad C = \gamma_i U$$

From (6.10a) we can also define the contact probability for a firm (γ) as

$$(6.11) \quad \gamma = C/n = \gamma_i U/n$$

Matchings are now simply defined as the number of successful contacts. In the infinite vacancy case $M = C$ since all contacts are by definition successful. In the single vacancy case $M = (1-z)C$ since z defines the probability of an unsuccessful contact.

We shall subsequently show how z depends on the number of firms in equilibrium, in which case our matching technology is simply a special case of equation (6.10).

We now have all the components of essentially two equilibrium models of search allowing for vacancies to open up without limit as individuals accept jobs or to be 'used up' so that other job searchers face a probability of null offers.

6.6. Equilibrium: The Unlimited Vacancies Case

The choice of variables to solve for in equilibrium is somewhat arbitrary since simple identities link some of the endogenous variables identified in the infinite vacancies model of search. It should be clear from inspection that all the behavioural relationship within this model can be written in terms of w^* , e and n and we will

therefore solve for these variables. The following relationships should therefore be noted

$$(6.12) \quad \gamma = e\mu$$

$$(6.13) \quad U = (N - ne)$$

In order to determine the equilibrium values of w^* , e and n we need three determining equations.

The determination of the firm's monopsony wage offer has already been considered and this has been shown to depend only on exogenous parameters of the model. The actual wage paid, however, may take into account 'bargaining' between worker and firm. This is possible because when a job searcher and firm meet in a search context they jointly have a surplus to distribute that arises because both will lose out by failing to engage in production. The cost for the individual is of course a search cost, for the firm it takes the form of unemployed capital.

To formally model this bargaining game that arises in a search context, the threat points of each of the players must be examined and then some rule for selecting from amongst the feasible wage bargains described. Diamond (1982) pursues exactly this course. Here, however, we seek a simpler formulation which nevertheless captures the essence of a bargained as opposed to monopsonistically determined wage. The essence of the bargaining solutions described by both Diamond (1982) and Pissarides (1984) is that wage payments reflect the strength of the two parties involved as essentially reflected in the level of unemployment. With high unemployment

individuals will expect a lengthy search before a job is found and firms will expect large flows of applicants. The converse is true when unemployment is low.

We therefore assume that in the presence of wage bargaining the wage paid will be determined by equation (6.14).

$$(6.14) \quad V_g = (b-c)p(U) + wp(E) = \alpha(U)$$

Equation (6.14) allows for individuals to derive a positive surplus from participation which is seen as being a function of the level of unemployment. As we have argued above the effect of an increase in U is to decrease α so that $\partial\alpha/\partial U < 0$.

This formulation is ad hoc in the sense that it might seem more appropriate to allow for individuals to secure a proportion of the total surplus generated by employment rather than a fixed sum. The problem with this is that if we allow free entry to determine vacancies the total surplus will be driven to zero. This problem does not arise in Diamond's (1982) paper because discounting is assumed, this forces a wedge between the profits of potential entrant firm (who must wait to receive applicants) and existing employing forms. As noted above (6.14) captures the essence of a bargaining solution without the need to view the complication of discounting. Equation (6.14) provides the first determining equation for equilibrium. Notice that w depends through U on equilibrium employment.

A natural choice for a determining equation for n is a zero profit condition. We therefore write

$$(6.15) \quad \pi_i = (\beta - w)e - k = 0$$

As firms enter the market the number of individuals contacting any one firm in any given interval declines, hence e declines and π_i falls. When π_i is identically zero there is no incentive for firms to enter and by the assumed symmetry of firms no incentive to exit.

Finally, we require a condition determining employment in equilibrium. Here we use the fact that in steady state inflows to employment must balance outflows so that employment in steady state satisfies:

$$(6.16) \quad \gamma_1(N - ne) - \mu ne = 0$$

The left hand term of equation (6.16) is the number of matchings (equals number of inflows) whilst the right hand term is the number of separations.

Equations (6.14) - (6.16) form a set of non linear simultaneous equations determining n , e and w . It should be apparent from inspection that the actual structure of these equilibrium equations for the 'infinite' vacancies model is rather simple, and therefore that many of the features of equilibrium can be deduced simply from inspection.

First consider the case where $\alpha = 0$ for all U so that $w = w^*$ which is determined solely by the exogenous parameters of the model.

It is now clear that equation (6.15) can be solved for e purely in terms of parameters whilst equation (6.16) can be solved for n in terms of e . We therefore obtain an explicit solution of the form

$$(6.17) \quad w = w^* = \frac{(\gamma_i + \mu)b - (b-c)\mu}{\gamma_i}$$

$$(6.18) \quad e = \frac{k}{(\beta - w)}$$

$$(6.19) \quad n = \frac{\gamma_i N}{e(\gamma_i + \mu)}$$

Equations (6.17) - (6.19) have a recursive structure that enables the effect of changes in exogenous parameters in steady state equilibrium to be immediately worked out.

We summarise these effects in Table 1 below

	b	c	k	β	μ	γ_i	N
w	+	+	0	0	+	-	0
e	+	+	+	-	+	-	0
n	-	-	-	+	-	+	+
ne	0	0	0	0	-	+	+

Table 1

Table 1 is worthy of detailed attention since it contains information that fully characterises the determinants of search equilibrium for the infinite vacancies or unrationed offers model of search.

Starting with an increase in the utility of leisure b we see that this increases the participation wage directly and hence under the assumption of $\alpha = 0$ also the market wage. With the necessity to pay higher wages profits fall and some firms exit the market $(n+)$, with this loss of firms the remaining firms receive a greater flow of applicants so that the steady state employment at each firm (e) increases. The effect on total employment of a change in b is zero. Employment is neutral to many changes in this model and to see why we refer back to equation (6.18). This shows that total employment (ne) is determined only by the parameters γ_i , N and μ and is independent of the endogenous determination of w . Hence changes in e and n must in most cases simply offset each other. Employment can be seen in this model as technologically determined, essentially by the matching technology and the separation rate μ .

The effect of an increase in search costs is of course identical to that of an increase in b ; the above comments therefore continue to apply.

If we consider an increase in the productivity of labour β we can see that in the absence of worker bargaining power over wages the effect on the market wage is identically zero. With greater productivity and unchanged wage payments profits will increase inducing entry of firms $(n+)$. With more firms in equilibrium each firm will receive a smaller share of total applicants so that e falls.

Again the total employment effect of a change in β is identically zero.

An increase in capital cost k is analogous to a decrease in productivity and the above comments apply in reverse for the parameter k .

The parameters γ_i and μ are respectively the rates at which firms and unemployed individuals meet and separate. Intuitively we should expect increased matchings to result in greater employment in steady state and vice versa for separations. This indeed is the case. Because the participation wage incorporates an element of compensation for costly search, the employment effect of these parameters feeds through onto wages.

In the above analysis we have assumed that an equilibrium exists. Existence within this simple model is an issue easily examined. The simplest way to see this is diagrammatically by graphing the equilibrium conditions (see Fig. 2)

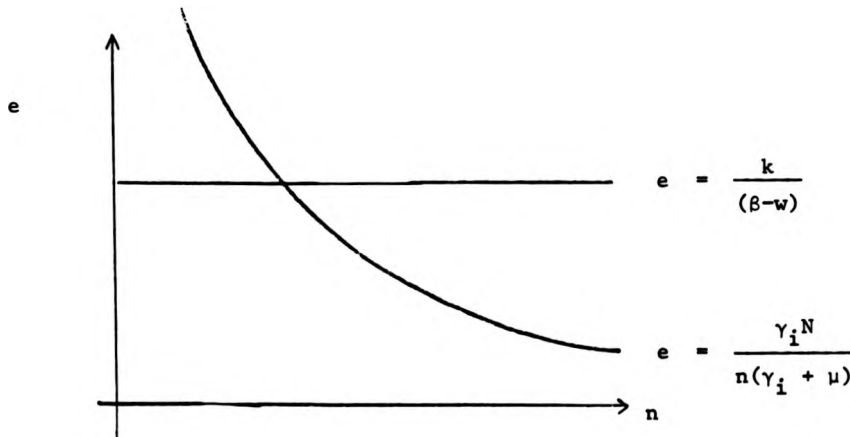


Figure 2

Provided a positive solution for w^* exists and provided that $w^* < \beta$, an equilibrium will exist and furthermore will be unique. This is so because e is a constant, whilst n is a continuous function of e on the interval $(0, \infty)$ with negative slope. A unique intersection in the positive quadrant is ensured.

So far we have made the simplifying assumption that $\alpha = 0$; what happens when we allow for worker bargaining power? Technically we lose the simple recursive structure of the determining equations alluded to above and now need to solve simultaneously for e and w . Without loss of generality we can set $b = 0$ (the effect of b is entirely analogous to that of c) and rewrite our solution as

$$(6.20) \quad w = \frac{c\mu}{\gamma_i} + \frac{\gamma_i + \mu}{\gamma_i} \cdot \alpha(U)$$

$$(6.21) \quad e = \frac{k}{\beta - w}$$

$$(6.22) \quad n = \frac{\gamma_i N}{e(\gamma_i + \mu)}$$

Simple inspection of equations (6.20) - (6.22) reveals that introducing the possibility of bargaining has no substantial effect except on wage determination. This is because employment remains determined in equation (6.22) by the search technology. Hence there exists a unique solution to equations (6.20) - (6.22) which can be determined recursively. Once the equilibrium employment level (n_e) has been solved for from (6.22) the wage can be determined from (6.21) (using the fact that $U = (N - n_e)$). The wage can then be used in equation (6.21) to solve for equilibrium steady state

employment in each firm and hence (using (6.22)) the number of firms in equilibrium.

The comparative statics of equilibrium are summarised in Table 2.

	c	k	β	μ	γ_i	N
w	+	0	0	?	?	-
e	+	+	-	?	?	-
n	-	-	+	?	?	+
ne	0	0	0	-	+	+

Table 2

The first three columns of table 2 are identical to those of table 1. When, however, we consider the consequences of changes in the parameters μ and γ_i the effect of wage bargaining is to make ambiguous our results.

To see why this is so, consider an increase in the leaving rate μ . From equation (6.22) this unambiguously reduces employment (ne), therefore unemployment rises. The effect of this upon wages is hence ambiguous. Higher unemployment certainly leads to an increase in the 'participation' wage but there is a countervailing effect through unemployment on workers' bargaining. Since the wage effect of an increase in μ is ambiguous, so from (6.21) is the effect upon expected employment at each firm.

Turning to the last column of table 2 the effect of an increase in the number of individuals participating can now be thought through. As N increases so (from (6.22)) does employment. However, calculating the rate of change of employment with respect to N reveals it to be $\gamma_i/(\gamma_i + \mu)$ which by definition is less than unity hence unemployment rises. Higher unemployment leads to lower wages (through the effect

on bargaining) and hence entry of firms and a lowering of each firm's level of employment.

So far we have described the physical aspects of equilibrium in our infinite vacancies model; what are the efficiency properties? In order to examine the efficiency issue and also to cast further light on the nature of the outcome we have described it is useful to consider the total value to all agents involved of the operation of the labour market we have described.

The flow of total value can be written as

$$(6.22a) \quad TV = (\beta - b)ne - (c - b)(N - ne) - nk$$

Steady state employment results in an expected value of output of βne . In producing output an expected number of ne individuals lose the value of their leisure (b). A further expected number ($N - ne$) are unemployed and enjoying leisure but incurring search cost c . A total of n firms are incurring flow costs of capital k . As a social planner concerned with maximising the money value of output net of all relevant costs and ignoring the distribution of gains between workers and firms could choose n - the number of firms operating to maximise (6.23) subject to the steady state employment condition given by (6.22).

This particular programming problem has a very simple solution, however, since whatever n is chosen equation (6.22) reveals that total employment is fixed. Graphing TV against n therefore reveals a downward sloping linear relationship as in Figure 3.

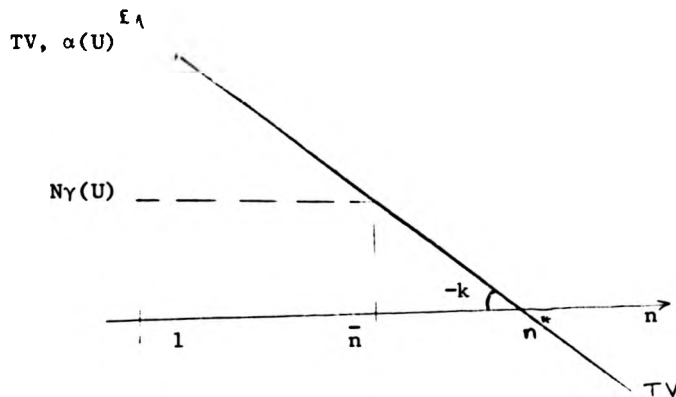


Figure 3

This reveals the artificial nature of the infinite vacancy assumption and also proves useful in considering the welfare consequences of search equilibrium.

First note that the optimum number of firms is simply 1. This arises because as equation (6.22) shows employment in steady state is independent of the number of firms. The best that can be done therefore is to minimise the capital payments required to generate the given employment level. This is not a surprising result given our assumptions on the nature of vacancies in the infinite vacancy world - they have zero marginal cost.

What of the unregulated market outcome then? If workers possess no bargaining strength so that $\alpha=0$ for all U , then the zero profit condition (6.21) dictates that n^* firms will exist in equilibrium. Entry will occur until TV is driven to zero.

If individuals bargain for wages according to the prevailing rate of unemployment then \bar{n} firms constitute an equilibrium. In this case the zero profit condition implies that $n\pi = 0$ where $n\pi = TV - NV_s = TV - Na(U)$. Since unemployment along with employment is technologically determined $Na(U)$ is constant as drawn in Fig. 3.

In our infinite vacancy setting worker bargaining is always (to a limit) a good thing, giving rise to higher welfare and affecting employment. This is precisely because firms are not required to increase vacancies when vacancies within any one firm are unlimited. What is useful about the model we have described here is that it provides a base against which to compare the more realistic null offer model.

We have shown that in an infinite vacancy setting a unique employment equilibrium exists and is characterised by employment independent of the demand price of output, cost of capital or search costs. The outcome is not affected by the existence of worker bargaining except that bargaining serves to reduce excessive entry and hence improves welfare. Search is always successful in such a setting; a job once accepted does not reduce the stock of jobs waiting for unemployed searchers. These last features are the essential distinction between the infinite vacancy world and the null offer world that has been subject to partial analysis in earlier chapters. What are the consequences therefore of allowing for null offers on the structure of search equilibrium?

6.7. Equilibrium: The Case of Null Offers

In order to examine the equilibrium of the single vacancy firm model of search, we simply need to rewrite the determining equations with appropriate substitutions made.

In place of steady state employment at each firm (e) we now have the proportion of time that each firm spends with its vacancy filled. We can therefore consider the number of firms to be determined by a zero profit condition.

$$(6.23) \quad \pi_s = (\beta - w)z - k = 0$$

For clarity we will henceforth use z to denote $(1-q)$ the null offer probability (q is the probability of obtaining an offer).

Wage payments can again be considered to be determined by wage bargaining subject to the complication that the availability of offers (q) enters into the determination of the participation wage.

Using the fact that $q = (1-z)$ we have

$$(6.24) \quad w \text{ s.t. } V_s(w) = \alpha(U)$$

$$\text{c. } (V_s(w^*) = 0)$$

Finally we need an analogue of equation (6.16) to determine steady state employment. Straightforwardly we therefore write

$$(6.25) \quad \gamma_i(N - nz) - \frac{wnz}{(1-z)} = 0$$

(6.25) takes into account the fact that of total contacts, only a proportion $(1-z)$ ($=q$) will be successful.

Equations (6.23) - (6.25) form a set of non linear simultaneous equations fully determining a search equilibrium in which n firms spend of a proportion of time z with a single vacancy filled and where $(1-z)$ denotes the probability that search is successful.

Again we start by abstracting in the first instance from wage determination. Since with offer rationing the availability of offers determines in part the minimum wage offer that will induce participation we need to impose stronger assumptions to exogenise wages. It was noted in section (6.3) that equating search costs to zero ensured that $w^* = b$, we therefore assume in the first instance that $\alpha = c = 0$.

We can rewrite the determining equations (6.23) - (6.25) as

$$(6.26) \quad w = b$$

$$(6.27) \quad z = \frac{k}{(b-w)}$$

$$(6.28) \quad n = \frac{\gamma_i N}{(\gamma_i z + \frac{\mu z}{(1-z)})}$$

With exogenously determined wage the null offer model is formally identical to the infinite vacancies model. This should not be surprising since the only substantive role for offer rationing lies in its determination of individual behaviour (participation). In the absence of any feedback onto firms' profits the only difference between the 'null' and 'unlimited' offers models lies in the technical condition for steady state employment equilibrium.

We can again establish that provided z lies within a feasible range ($0 \leq z \leq 1$ by virtue of being a probability) then equilibrium exists and is unique. Equation (6.28) inverts to solve for z as a single valued monotonically decreasing function of n and therefore Fig. 2 is again an appropriate diagram. If $k > (\beta - b)$ then no equilibrium exists.

The comparative statics of this simple equilibrium are (excepting the exogenous wage) exactly the same as for Table 1.

What happens when we allow for offer rationing to feed back (via wages) on profits and therefore on entry incentives? To consider this, allow $b = \alpha = 0$ but assume that $c > 0$. Setting b to zero again involves no loss of generality, assuming costly search makes certain that offer rationing matters. Again for simplicity we set worker bargaining power at zero.

We now have the following equation determining the equilibrium wage.

$$(6.29) \quad w = \frac{c\mu}{\gamma_i(1-z)}$$

Equations (6.23), (6.25) remain valid for the determination of n and z .

With this slight modification of our model an interesting possibility is opened up. z must now solve a quadratic function so that we have the possibility of multiple equilibria.

Diagrammatically,

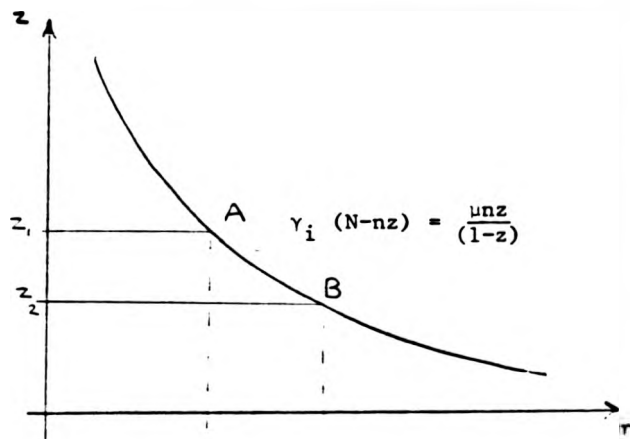


Figure 4.

z_1 and z_2 represent the possible positive roots of

$$(6.30) \quad -\beta z^2 + (k + \beta - cu)z - k = 0$$

It will be noticed that in the unlimited vacancy model the assumptions made here would ensure a unique equilibrium. More important, however, than non-uniqueness is the fact that the multiple equilibria described here for the null offer model will in general entail different levels of employment. Non uniqueness results from the fact that wages reflect the extent of offer rationing. We have the possibility of an equilibrium with considerable risk of null offers (high z) which therefore requires a high wage payment to encourage participation which in turn discourages the entry of firms to create more vacancies. However, if more firms were to enter then the probability of a filled vacancy declines as does the

equilibrium wage. Another equilibrium may exist again with zero profits, a large number of firms and only a small probability of being unsuccessful upon randomly contacting a firm.

In Figure 4, A corresponds to the first equilibrium possibility which we might call a tight labour market (high null offer probability, small number of vacancies), B corresponds to the second possibility, a 'loose' labour market.

It is not clear which equilibrium of A or B represents greater employment since total employment is given by nz . We can, however, exploit our equilibrium conditions further in order to resolve this issue.

To do this we start by noting that there must be consistency between an individual's unemployment experience and aggregate employment. This consistency is expressed in the identity (6.31)

$$(6.31) \quad p(U) \equiv \frac{U}{N} \equiv \frac{(N-nz)}{N}$$

The proportion of time that each individual spends unemployed must represent a 'fair share' of total unemployment.

Using this identity the participation wage can be re-expressed as a function of total employment rather than simply offer rationing. We therefore obtain ($b = 0$)

$$(6.32) \quad w^* = \frac{cp(U)}{p(E)} = \frac{c(1 - (\frac{n}{N})z)}{(\frac{n}{N})z}$$

The participation wage is therefore decreasing in total employment since greater aggregate employment means less individual unemployment.

We can, however, use this fact to deduce that unemployment is higher in that equilibrium associated with the highest wage. In other words the tighter labour market (equilibrium A) is associated with higher unemployment, higher wages, more rationing of offers and fewer firms.

By allowing in the simplest possible way for a constraint on vacancies in a model of search we have opened up the possibility of a genuinely interesting multiplicity of equilibria. Where two equilibria are allowed one may be associated with lower employment and greater rationing of offers than the other. This bootstraps feature of search equilibrium is absent from a model that ignores offer rationing because in such a model there is no feedback from rationing onto profits and therefore entry. Even where in the unlimited vacancy model we endogenised wages we saw that even multiple equilibria were associated with a unique employment level.

The existence of multiple equilibria of course complicates the comparative statics analysis of outcomes. It is clearly possible that the two different equilibria have completely different and possibly opposite comparative statics properties. However, by restricting attention to only stable outcomes we are nevertheless able to carry out a comparative statics exercise.

Our three determining equations are (6.27) - (6.29) which for compactness we will denote f_1, f_2, f_3 respectively. The effect of an increase in some parameter θ on the equilibrium values of w^*, z and n can be ascertained from a consideration of the differentials of the three equilibrium conditions, since before and after the change in θ these must be satisfied. Hence

$$\begin{pmatrix} df_1 \\ df_2 \\ df_3 \end{pmatrix} = J \begin{pmatrix} dz \\ dn \\ dw \end{pmatrix} + \begin{pmatrix} f_{1\theta} d\theta \\ f_{2\theta} d\theta \\ f_{3\theta} d\theta \end{pmatrix} = 0$$

Where J is the Jacobian Matrix of the determining equations. The sign of the effect upon the endogenous variables of an increase in θ is therefore given by

$$\text{sign} \begin{pmatrix} dz/d\theta \\ dn/d\theta \\ dw/d\theta \end{pmatrix} = \text{sign } J^{-1} \begin{pmatrix} f_{1\theta} \\ f_{2\theta} \\ f_{3\theta} \end{pmatrix}$$

The Jacobian of our three equation system has a sign pattern given by

$$J = \begin{pmatrix} - & 0 & + \\ - & - & 0 \\ - & - & - \end{pmatrix}$$

If we restrict attention to a stable equilibrium then by the Routh conditions (see Chiang(1974)). the determinant of J is positive.

The comparative statics properties of a stable equilibrium are therefore summarised in Table 3.

	c	β	k	μ	γ_i	N
w	+	+	-	+	-	0
z	+	-	+	+	-	0
n	-	+	-	-	+	+
nz	-	+	-	-	+	+

Table 3

Table 3 indicates that not only does rationing imply non unique employment equilibria but also that any equilibrium is non neutral (vis a vis employment) to changes in the exogenous parameters.

The sign pattern of Table 3 corresponds closely to that of Table 2 so that we will not repeat an explanation of these effects. The main differences between Tables 2 and 3 concern the employment effects which we will detail below and the wage effects of changes in β and k in the null offer case.

These latter effects arise exactly because of the employment effects that have been described. Taking for example an increase

in the value produce (β) of labour, it can be seen that the direct effect of this is to increase profits. Subsequent entry therefore reduces the degree of offer rationing and there follows a reduction in the participation wage. The overall effect upon the wage is hence negative.

Turning now to the employment consequences of parameter shifts it can be seen that changes in c , β and k are no longer neutral with respect to employment.

In the case of an increase in search costs the effect of an increase in the participation wage is to decrease profits. Decreased profits in turn cause exit which in turn causes z to increase. This latter increase in rationing reinforces the initial effect on the participation wage so that the above process is repeated. The increase in z in equilibrium is not sufficient to offset the loss of firms and employment is lower. The new equilibrium is therefore characterised by lower employment, more rationing of offers and higher wages. This then is a 'general' equilibrium refutation for the usually suggested partial result of the consequences of an increase in search costs. In this model, however, the effect is complicated, since all agents always participate any employment effect comes through entry of firms rather than increased reservations.

Once again a welfare analysis is most easily made by considering the aggregate total value given in equation (6.22a) (replacing e with z). In order to understand the null offer model it is necessary to calculate how total expected employment (nz) varies as a function of the number of firms in equilibrium. To see this consider once again equation (6.28). Equation (6.28) can be regarded as defining

employment (nz) as an implicit function of the degree of offer rationing (z). Differentiation of this relationship reveals that $dnz/dz < 0$ which in turn implies that $dnz/dn > 0$. This is a sensible result since it implies that the entry of firms decreases unemployment. As firms enter of course the rate of contact by unemployed workers declines but the total effect of entry on employment is positive. We have shown above that zero profit equilibrium is consistent with two possible values of z and hence of n . We can therefore deduce the shape of total profit (which in the absence of bargaining equals TV) as a function on n . This is drawn below in Figure 5.

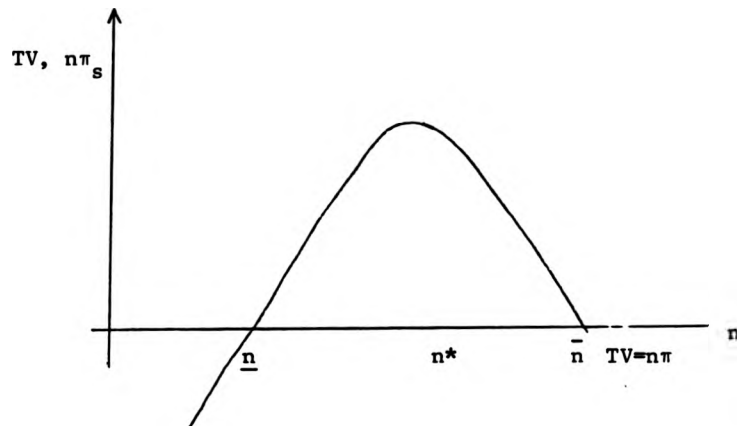


Figure 5

The zero profit condition implies in the absence of worker bargaining two equilibria, shown in the diagram above as \underline{n} and \bar{n} . We have demonstrated earlier that \bar{n} is associated with higher total employment, a lower (participation) wage and a lower null offer probability (z).

Both equilibria are identical in welfare terms and both inferior to the social planners best choice indicated by n^* . In the

case of \bar{n} there is too much entry and too little unemployment. All agents could potentially be made better off if one firm were to quit the market and the saved capital payments redistributed amongst existing firms and individuals. In the case of \underline{n} the opposite is true, all could gain by the entry of an additional firm and an increase in steady state employment provided some system existed for lump sum transfers out of the extra value product of output (β).

Figure 4 can also be used to give some intuitive feel for the implicit dynamics of the equilibria. Whenever total profits are positive there is an incentive to enter so that n increases, if total profits are negative then we would expect firms to exit. According to this dynamic therefore the \bar{n} equilibrium is stable (over the range of $n > \underline{n}$ $n < \infty$) whilst the \underline{n} equilibrium is globally unstable. Hence figure 4 helps us to deduce that our comparative statics exercise reported above refers to the \bar{n} (high employment) equilibrium. Indeed to confirm the comparative statics analysis one simply has to shift the TV curve in Figure 4 and examine the consequences for \bar{n} and then use equations (6.27) and (6.28) to deduce the consequences for z and w^* .

So far then by considering the single vacancy null offer model we have deduced an interesting possibility. Even in the most simple circumstances there exists the possibility of multiple employment equilibria. The same set of parameter values may be consistent with both a high unemployment or low unemployment outcome. However, both equilibria generate the same level of total welfare for the agents involved and only one (the high employment equilibrium) is stable. Both equilibria are inefficient because firms act as

Nash agents calculating the profit to be gained if they enter assuming that the effect of their entry on market outcomes is negligible.

Allowing for workers to bargain for a part of the total surplus allows some of the more questionable features of our model highlighted analysis to be resolved.

Firstly, it should be noted that whereas above unemployment and wages are positively correlated (since wages represent a minimum compensation for the costly search necessitated by participation) this correlation is broken once bargaining is considered. Our wage equation (6.14) allows for wages to increase as unemployment falls and workers are better able to bargain with firms.

Secondly the welfare implications of equilibria are altered. The effects of wage bargaining can most easily be seen using Figure 5. If we draw the total value to workers on Figure 5 we obtain an upward sloping relationship as in Figure 6.

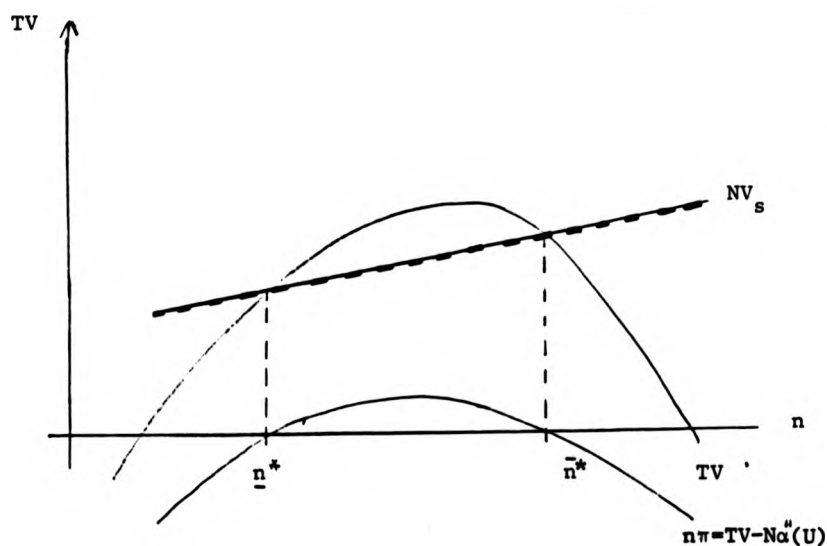


Figure 6.

The NV_s schedule is upward sloping because of the relationship between n and unemployment ($N-nz$). Equilibrium is now not given by the intersection of the TV schedule with the n axis. Using the fact that $TV = NV_s + n\pi$ zero profit entry requires that $TV - NV_s = 0$ so that our multiple employment equilibria are \underline{n}^* and \bar{n}^* . It is now clearly the case that \bar{n}^* implies a higher level of welfare than the low employment equilibrium \underline{n}^* . Hence bargaining for wages may give rise to more desirable equilibria and may also imply that multiple equilibria have different welfare properties. As drawn in Figure 6 the high employment equilibrium is again stable and save for the fact that the wage is indeterminate table 3 can be used to examine comparative statics properties.

In drawing Figure 6 the value accruing to workers is shown as a linear function of n , even though U is a non linear (concave) function of n . If we relax this unrealistic linearity and simply require $N\alpha(U)$ to be monotonically increasing in n (though the fact that U is monotonically decreasing in n) then the equilibrium possibilities multiply. Figure 7 shows one possibility.

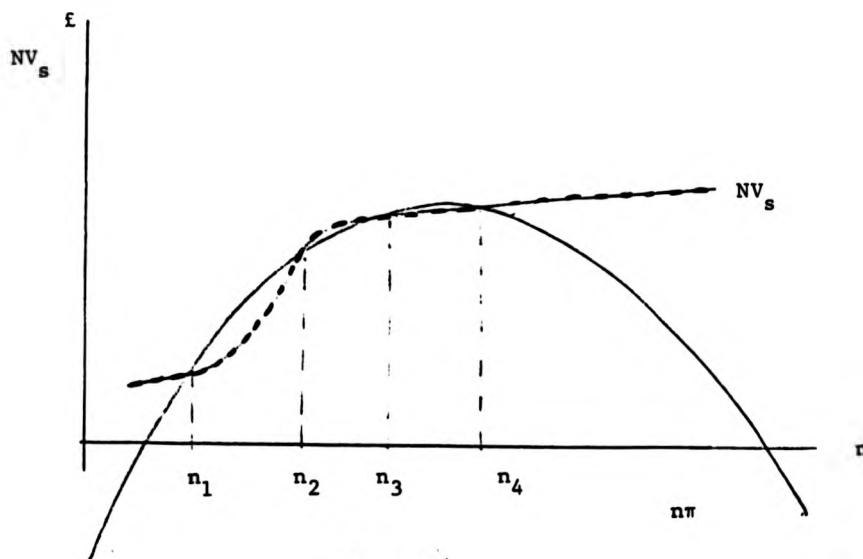


Figure 7

With a highly non linear bargaining function the possible equilibria proliferate. The intuition behind this is clear. Previously (when $\alpha = 0$ for all U) the only source of feedback from market outcome onto firms entry arose through the participation decision of individuals. High unemployment necessitated large wage payments to encourage participation. Now with bargaining there is a second avenue of feedback that complicates the picture, unemployment affects the bargaining position as well as the participation decision. If this feedback is appropriately non linear the multiplicity problem worsens.

As drawn in Fig. 7, there are four possible equilibria n_1 with the lowest employment and lowest welfare through to n_4 (highest employment and welfare). Whilst it is still true that n_1 is unstable and n_4 a stable outcome it is now possible to have an equilibrium which is inefficient compared with alternative and stable. Starting from n in the interval $n_1 < n < n_3$ the implicit dynamic process of entry leads to n_2 as an equilibrium.

Allowing for wage bargaining in the null offer framework has therefore opened up the possibility of a genuinely interesting multiplicity of equilibria. We have shown how stable inefficient equilibria might arise even in a world characterised by a very simple search technology and production process. Equilibria entail different employment outcomes, different null offer probabilities (which are negatively correlated with employment) and different wages (which may show no particular correlation with employment).

These possibilities can be contrasted with our analysis (assuming the same search and production technology) of the infinite vacancy world which allowed for only successful search. Employment was uniquely determined in such a setting as a function of only the search process.

6.8 Concluding Remarks

In this chapter we have drawn together the component parts introduced in previous chapters and formulated an equilibrium model of search with offer rationing or null offers. In considering an equilibrium model of search we have trodden new ground; the analysis of search equilibria is in its infancy.

The main purpose of this chapter has been to compare and contrast models of search where offer rationing is or is not a feature and to that end we have formulated the simplest models that seem to capture the appropriate notions of vacancy creation. It has been seen that an essential difference implied by the existence of constraints on offer creation is with regard to the uniqueness of employment in equilibrium. In the case of unrationed vacancies employment was technologically determined by our simple matching assumption. Employment was independent of all parameters save those directly affecting the matching process. It is important to note that this conclusion is not robust to different specifications of matching technology, indeed the work of Diamond (1982) and Pissarides (1984) confirms this. However, as assumptions go our assumption was not a bad one. Contacts were seen as being initiated by unemployed searchers so that total contacts were independent of the number of firms in equilibrium. This simplifying restriction on matching processes we feel at least worthy of testing or empirical investigation. In any case, it remains an interesting question

whether assumptions that give rise to uniqueness and neutrality in a model with limitless vacancies continue to give these conclusions when offer rationing is allowed.

We have in fact seen an important distinction between rationed and unrationed models. When offer constraint was allowed it has been shown that steady state employment once again depends upon all the parameters of the model. Indeed, we have shown that simplistic partial equilibrium results concerning the effect of increased search costs for example are reversed in an equilibrium setting. Higher search costs imply higher wages, fewer firms, greater rationing and lower employment. More importantly, however, rationed equilibria are non unique. There exists the possibility that the market can be in equilibrium with high unemployment and wages when exactly the same parameters are consistent with a low unemployment equilibrium. The problem exists because offer constraint determines the wage required to induce participation which in turn partly determines profit and thereby entry incentives. High wages, high unemployment and 'rationed' vacancies can therefore persist as an equilibrium in circumstances where if firms could be induced to enter a low unemployment, zero profit equilibrium is possible.

We believe this non-uniqueness of equilibria is the most important insight to come out of an equilibrium model of search with null offers. Inefficiencies of Nash wage bargains will exist whether offers are scarce or effectively infinite, bootstrap equilibria, however, arise out of the very nature of offer rationing and the feedback on vacancy creation implied by it.

From this point of view it should be noted that the simplifying assumptions made in the course of analysis add strength to the results. We deliberately restricted attention to a very simple matching technology which allowed for total matching to be a linear function of unemployment. If a more general matching function is allowed for it follows that the multiplicity of equilibria result will be reinforced. Furthermore by introducing worker bargaining into the simple framework discussed here we have established that stable inefficient equilibria exist. These equilibria are inefficient not only with regard to the social optimum but also in comparison with other market outcomes. This distinction is important since social optima may not be obtainable (depending upon the ability to make the appropriate lump sum transfers to keep firms in business) whereas market outcomes are certainly feasible. The existence of the kind of inefficiencies discussed in this chapter offer a rationale for intervention which does not depend upon precise welfare calculations but simply upon the desire to shift the 'natural rate' of unemployment to a more desirable level.

Finally, it should be noted that the equilibrium models discussed here are of the 'vacancy' search kind discussed in chapter 2. Equilibrium consists of a unique wage offer made by all firms. It is, however, possible to use the techniques discussed here to model search equilibria with wage dispersion. Indeed, the simplified form of the models developed in this chapter makes a model that generates wage dispersion analytically tractable in a way not possible with the work of Diamond (1982). A necessary condition for wage dispersion is that searchers' reservations differ. Clearly a

continuous distribution of reservations (via a dispersion of search costs) cannot easily be handled. However, a two-point discrete distribution can easily be allowed for. Equilibrium may then consist of a proportion of firms paying low wages which only high cost (low reservation) searchers accept whilst the remaining firms make offers that all (high and low cost) searchers find acceptable. It is not to be expected that the efficiency or non uniqueness questions posed by this chapter will significantly differ in these new circumstances. The questions which one may ask concerning such a model are the determinants of the degree of wage dispersion. Again this issue is left to future work.

Chapter 7

Conclusions

7.1. Summary and Discussion of Results

This thesis has attempted to analyse the consequences and causes of offer rationing in markets characterised by job search. The previous chapters can be divided into three headings according to the focus of investigation. The results of the analysis are considered below, in line with that division.

a) The Consequence of Null Offers for Individual Decisions

In chapters 2 and 3 the consequences (in a partial equilibrium sense) of offer rationing in a search environment were discussed. Chapter 2 provided a somewhat restricted survey of the literature, the results of which are well known and hence will not be discussed further. Chapter 3 started from a belief that learning by individuals is an interesting and important phenomena. There is no empirical evidence to support this but considerable casual evidence (see Hey (1981)) that individuals are not fully informed as to the circumstances in which they search. Imperfect knowledge was limited to the parameter of offer rationing and a number of results were demonstrated. In situations where bygones are bygones (and past offers cannot be revisited) it was shown that imperfect information may be a discouragement to search, a result which runs counter to most results concerning the effects of imperfect knowledge. The reason for this reversal stems from the inability to return to past offers because where 'recall' is allowed it acts rather like insurance and thereby encourages more search. There are two kinds of application for this sort of study. Firstly imperfect knowledge suggests one way in which 'optimism' or pessimism might enter into economic models. It is not necessary for agents to hold false expectations nor even be risk averse for economic policies which act upon beliefs about the future to be effective. Merely making individuals more certain about the future may have payoffs.

Secondly it is desirable that attempts at empirically implementing models of individual search behaviour should incorporate at least the possibility of learning effects. It is of course very difficult to think of a suitable proxy for as abstract a notion as 'diffuseness of priors' but degree of belief in predictions is something that may be available.

Taking the first possible application of our results in chapter 3, one can think of macroeconomic models. A popular held belief associated with the development of the rational expectations hypothesis concerns the neutrality of many kinds of macropolicy intervention. It is argued that if individuals know how the economy works and condition their expectations on such knowledge, any government policy which relies on only changing the beliefs of individuals will fail. The 'Adaptive' model of search decisions analysed in Chapter 3 points to one deficiency in this kind of argument. Even if individuals form expectations of the effect of policy that are unbiased, the degree of belief in those expectations might also be important. This is so in an 'adaptive' situation even if agents are risk neutral. Hence any policy which simply confirms expectations and means that individuals hold their beliefs with greater certainty in the future, may well have real effects. Simply confirming the state of the labour market (as proxied by q in our notation) might be sufficient to encourage greater economic activity (participation).

Turning now to the second application, we believe that the model of chapter 3 has most direct relevance to the issue of worker discouragement. In the absence of learning the idea of a discouraged

worker is more closely related to the idea of an idle machine. Some economic environments are sufficiently unrewarding as to lead to an optimal decision of idleness. The word discouragement, however, implies a process rather than a state and it is that process that the analysis of chapter 3 captures. Individuals may become idle even when circumstances dictate a positive net benefit to participation simply because of a run of bad luck. Bad luck in chapter 3 took the form of a sequence of unsuccessful searches. Crucial therefore to an explanation of worker discouragement is the notion of adaptive learning.

b) The Nature of Wage Payments and the Causes of Null Offers

Unlike the literature on individual search decisions the literature on the decisions of firms in a job search context is very limited. The first job in chapters 4 and 5 was therefore to outline a framework of analysis that would provide some insight into the determinants of wage offers and vacancy creation. This job was therefore one of model specification. In order to allow for some generality as regards vacancy creation it was necessary to make fairly strong assumptions regarding both the objectives of firms and the environment in which they operated. Where possible (i.e. in special cases) the importance of these assumptions was, however, considered.

Chapter 4 dealt with the determination of wages and it was argued that the only notion of wage determination consistent with job search involved monopsony or bilateral monopoly bargaining. Given this, the profit maximising choice by a firm over its wage offer was considered. It was demonstrated that even in a very

simple setting the existence of a dispersion of reservation wages causes considerable difficulty for analysis. The response of monopsony wage offers to changes in job search parameters is largely ambiguous when the firm has more than a single vacancy to fill. The important point to note about this result is that it rules out any notion of wages performing the task of eliminating excess demand or supply. An increased flow of applicants might result in an increase in wage offers thereby increasing the incentive to enter the market and search. Only in one special case were wages easy to determine and this was when vacancies were unlimited. In this case wages were totally invariant to labour market conditions. Since unlimited vacancy creation entails an assumption of costless vacancies this special case is really of little interest. In general the monopsony wage offers of firms respond in an ambiguous way to changes in labour market conditions.

Chapter 5 was concerned with the determination of vacancies and thereby of offer rationing. The analysis concentrated on a fairly appealing kind of technology that naturally gave rise to limited offers by firms, independent of any product market sales constraints. It was shown that vacancy creation depended in a fairly intuitive way upon labour market conditions. For example, an increase in the flow of applicants was consistent with an increase in the maximum labour force that a firm would employ. Vacancy creation was also related back to offer rationing and the possibility of a positive feedback to firms and individuals' decisions was a possibility. This offers some idea of the way in which a search equilibrium may be non-unique and display

bootstraps type features. The results of chapter 5 were shown to be reasonably robust. Where analytic methods failed to produce unambiguous answers numerical experiments were employed to check robustness. Chapter 5 did not consider a firm varying its wage offer with employment, but given state independent wages the analysis there suggested offer rationing as quite a general feature of markets characterised by search.

Chapters 2, 3 and 5 taken together offer some insight into the nature of equilibrium in markets characterised by search and null offers. Offer rationing can arise quite naturally out of a stochastic employment process. If expanding employment is costly it will never pay to be prepared to accept applicants for jobs without limit. Indeed, if labour is only productive in conjunction with some other inputs then a decision regarding, say, capital is in fact an implicit decision about vacancies. This relationship is formalised in chapter 5.

We have also seen in chapters 2 and 3 that wages (or their dispersion) and the extent of offer constraint are important determinants of both search and participation decisions yet it is the outcome of these decisions that feedback onto firms in the form of flows of applications. A more thorough analysis of the feedback mechanism constitutes our third area of research.

c. The Equilibrium Consequences of Null Offers

Taking as a starting point the analysis of previous chapters, chapter 6 instigated an investigation of quantity constraint and search equilibrium. A consistent equilibrium model of search and turnover poses considerable difficulties, therefore the first part of chapter 6 dealt with conceptual problems. Equilibrium in a search model that incorporates rationing is then contrasted with an infinite vacancy world. The important point to come out of this analysis makes rigorous the suggestion made in chapter 5. Offer rationing determines an individual's search decisions; in a simple model where all individuals are identical this is reflected in the 'participation wage' discussed in chapter 2. But the 'participation wage' is also a parameter of the firm's profit function and thereby determines the incentive foreentry. Multiple equilibria can exist in which one equilibrium entails high unemployment, a strong rationing constraint and high wages whilst another equilibrium entails lower wages and unemployment. In the absence of offer rationing there is seen to be no feedback effect between firms and workers' decisions and unemployment has a unique natural rate.

The model discussed in chapter 6 is highly simplified and capable of generalisation. For example searchers could be allowed to choose a level of search intensity. The central issues would, it is conjectured, remain the same. The results are robust to assumptions regarding the determination of wages with either wage bargaining or unilateral wage setting by firms consistent with multiple equilibria. The analysis of chapter 6 makes explicit the multiple equilibria possibility alluded to by Diamond (1982).

Upon a more thorough analysis of the multiple equilibria problem we were able to indicate that once wage bargaining is allowed for different equilibria have different welfare consequences. Furthermore there existed the possibility that undesirable equilibria were stable and could therefore persist. Hence inefficiency does not rest upon a comparison of market solutions to possibly unattainable social optima.

In one sense the simplifying assumptions made throughout chapter 6 add strength to the nature of the results. Complicating factors, such as multiple vacancy firms, non linear matching technologies or search intensity decisions can only make matters worse as far as the non-uniqueness problem of equilibrium is concerned.

In conclusion chapters 2 and 3 concentrate on the consequences of offer rationing, 4 and 5 on the causes and chapter 6 on both. We have seen that offer rationing 'matters' in that it affects individual search decisions, is consistent with a firm's profit maximising decisions and has important consequences for the nature of equilibrium.

7.2. Future Research

The stated purpose of this thesis was an examination of the consequences and causes of offer rationing in labour markets characterised by search. 'Consequences' were considered in a partial equilibrium sense in chapters 2 and 3 whilst 'causes' were the concern of chapters 4 and 5. A market equilibrium view of both causes and consequences was offered in chapter 6.

The reasons why offer rationing and search should be an interesting area of analysis for an economist have yet to be discussed, in such a discussion lies suggestions for further work.

Search theory does provide a theoretical view of labour market operation that is broadly consistent with observations about labour market functioning. The new view of unemployment as detailed by

Clarke and Summers (1979) notes that unemployment spells are experienced by many members of the working population, that unemployment spells are relatively short and therefore that the 'unemployed' turnover relatively quickly. This stock-flow emphasis on unemployment is hard to reconcile with standard (static) textbook labour market theory but does correspond well to the ideas of job search and job turnover dealt with in the preceding chapters.

Perhaps because of the views of the original contributors search theory most often assumes that job offers will always be forthcoming and that wage payments are the most important element in an individual's job choice decisions. As long as wages reflect the true (social) value of an individual's labour such assumptions are generally taken to imply an efficient labour market in which unemployment is efficient and productive. The optimising decisions of individual agents cannot be improved upon as long as these decisions are made contingent on correct prices.

It has been demonstrated in the preceding chapters that such a view, if not incorrect, then certainly needs to be viewed with caution once one allows for offer rationing or null offers. Such rationing does not arise only because markets fail (see chapter 5) and therefore our analysis is not limited to 'Keynesian' or any other particular view about product market clearing. The fact that agents' decisions interact through an offer constraint variable opens up the possibility of multiple equilibria which differ considerably as regards unemployment and efficiency.

There are many aspects of search and offer rationing that remain to be analysed. As far as individual search is concerned I am inclined towards the view that individuals are motivated by a great deal more than simply the search for better wages. An alternative view towards the motivation for search can be found by adapting Lancaster's (1979) approach to product variety. Jobs differ in many respects and wage dispersion might simply reflect compensating differentials. Individuals differ in their subjective evaluation of the trade-off of characteristics and therefore search for a good 'match'. This has the important implication that search is socially desirable provided that it results in better matching of agents to jobs. In such a setting offer rationing inhibits search and might result in searching agents crowding each other out of their most desired job locations (as casual evidence for this one can cite the increase in better qualified applicants to unskilled jobs in times of high or rising unemployment).

As far as firms' decisions are concerned, economic theory still needs to produce a theory of firms' behaviour which is consistent with attaching wages to the characteristics of jobs rather than the applicants for them. This is a pervasive and well documented phenomenon. Typically wages are not seen to respond to either employment conditions, the state of the labour market or to product demand conditions. The invariance of wages to product demand is the concern of the recently emerged (and still emerging) contract literature (see Hart (1983)). The idea is that firms insure workers against such fluctuations and this idea I believe will be instrumental in explaining the invariance of wages to labour market conditions and employment. In this explanation lies a more

convincing account of the failure of firms to increase employment by making lower wage offers. In our work, we have assumed 'sticky wages'; the causes of this phenomenon remain to be examined.

Finally much remains to be done in considering the consequences for market equilibria of search activity. The models so far discussed in the literature have provided considerable insight. The inefficiency of search equilibria is a pervasive phenomenon given the externalities involved in matching processes. I feel that some empirical work on the nature of real world matching processes and on the effects of job agencies (government or private) in search environments is absolutely crucial. All policy conclusions depend upon a more precise knowledge of these matching processes, without such knowledge economists must remain silent on important policy issues relating to search. On a wider point I also feel that work is urgently needed on equilibrium models of search that allow for the features described above, namely search motivated by a desire for better (and not just better paying) jobs and firms that pay wages independent of many prevailing economic conditions. In such an environment I would suggest that freely operating agents are unlikely to produce efficient outcomes, that the notion of involuntary unemployment again becomes meaningful and that costs of unemployment (in the form of mismatching) hitherto ignored become crucially important. Finally, work on the nature and causes of wage dispersion in markets characterised by job search is still required.

In short, much remains to be done.

Appendix : Computational Methods

In chapters 3, 4 and 5 numerical results were used in order to examine the effects of parameter changes on agents' behaviour where no analytic results were possible. The purpose of this appendix is to explain the techniques used.

A1 Adaptive (learning) Reservation Wages

The analytic derivation of the reservation wage for an individual facing an offer distribution $f(w)$ was dealt with in chapter 2. Unfortunately it is not possible to solve explicitly for r and therefore even in the full information case a numerical evaluation routine is needed. The form of the equation to be solved is

$$(A1) \quad \int_r^{\infty} x(r) dr = 0$$

Standard or Modified Newton (slope) methods can be used to solve (A1) for r provided that a numerical integration routine is used to provide function values. I am grateful to B. Bacon for providing a ready written 8 point Gauss routine

A straightforward recursion technique can then be employed to calculate information contingent reservation wages. In the case of search without recall

$$(A2) \quad r(y,z) = -c + \left[\frac{y}{y+z} \right] E \max(W, r(y+1,z)) + \left[1 - \frac{y}{y+z} \right] \max[0, r(y,z+1)]$$

Equation (A2) is derived in chapter 3, here the expected value of the Beta distribution is written explicitly. In order to calculate a set of information contingent reservation wages it is simply necessary to find starting values for the recursion.

For large values of y, z the residual variance of the Beta distribution is very small and 'adaptive' reservation wages approach those for the full information case. We used $y = z = 100$ as an initial starting point and calculated using (A1) the full information reservation wage. Recursion is thereafter straightforward.

Where recall is allowed equation (3.18) can be used as the basic recursion and the above procedure goes through as before.

To calculate a set of information contingent reservation wages using the above method requires 100^2 iterations and as many numerical integration calls. Using a Burroughs B6700 mainframe computer each run took approximately 40 seconds c.p.u. The actual program written to perform these calculations is included at the end of this appendix and forms one of a group of programs written by the author to facilitate analysis of individual search problems. Whilst a normal offer distribution $|f(w)|$ was assumed, any analytic distribution can be handled without difficulty. The parameter values for the offer distribution were scaled so as to avoid rounding errors that inevitably accumulate over a large number of iterations.

I would like to thank Mr. P. Fisher for assistance in mounting and running programs on the Burroughs B6700

A2. Firms Wage/Vacancy Decisions

Where analytic results were not possible in chapters 4 and 5 numerical experiments were employed. Since we wished to examine the choice variable of a well defined maximisation problem it was possible to use existing NAG routines in a simple program to provide results.

The
NAG routines called are EO4EBF, EO4CGF, details of which are contained in NAG documentation. These are straightforward maximisation routines.

Again, whilst an exponential distribution was used to provide illustrative results any analytic distribution could be handled.

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